On the "Jagiellonian compromise" – voting in the European Union

Edward Ratzer
5 Charlotte Rose House, Cheltenham GL50 2PB, UK
Email: ear23@cam.ac.uk

3 April 2006

Abstract

The proposed "Jagiellonian compromise" for voting within the European Union Council of Ministers is questioned. An alternative approach based on linear weighting by country population is proposed.

1 Introduction

In [1], a voting system is proposed for the European Union Council of Ministers. This system has received publicity via the physics press [2], the media (eg [3], [4]) and Parliamentary documents [5]. The system consists of two steps:

Weight allocation Weights are allocated to each country in proportion to the square-root of the country's population. The sum of all the weights are normalised to 1.

Voting threshold A weight threshold is found, such that motions in the Council of Ministers are passed if the sum of the countries' weights in favour exceeds the threshold. This threshold is calculated by finding a point where each country's voting weight is approximately equal to its voting power (as defined by the normalised Banzhaf index).

This paper will look at each of these steps in turn.

2 Weight allocation

The square-root weight allocation of [1] is derived by first looking at the probability that an individual voter can influence the result of a binary vote (eg a yes/no referendum). An individual voter has direct influence if the rest of the population (K) cast an equal number of yes and no votes. This approach requires the rest of the voting population to be even in number – otherwise an individual will never have a casting vote. Common-sense dictates that an individual does have some voting power in a population with even number. This non-uniform applicability casts doubt on this approach, however we will ignore this doubt in the following.

In [1], the probability of tie, p_{tie}, is derived by starting with the simplifying assumption that each member of the population is equally likely to vote in the two directions:

$$p_{tie} = \begin{pmatrix} K \\ \frac{K}{2} \end{pmatrix} \frac{1}{2}^K = \frac{K!}{\left(\frac{K}{2}!\right)^2 2^K} \approx \sqrt{\frac{2}{\pi K}} \propto \frac{1}{\sqrt{K}}$$

This result can be visualised by considering the width of the binomial distribution varying with its standard deviation ($\sqrt{K/4}$). The shape for large K approximates a fixed shape (a normal distribution) and has a fixed area: therefore the height of the centre point drops as $1/\sqrt{K}$. [1] therefore suggests that country voting weights be allocated in proportion to $\sqrt{N}$ (where $N = K + 1$ is the country's population) so that the proportions cancel out to give each individual in the Union an equal voting weight.

The assumption of equal probability of voting in each direction is not necessary. The probability $q(x) = \Pr(\text{probability each individual votes 'yes'} = x)$ can be marginalised out. A uniform prior is suggested for simplicity. This prior is flat, which could be seen as the opposite to placing all the the probability mass at a single point (eg $q(x) = \delta(x - 0.5)$ as in [1]). For the model of [1] with a population of 10 million, voting results between 49.98% and 50.02% with probability 0.95 would be expected. Such close votes are rarely seen and which gives support to an alternative prior.

Testing a whether the uniform prior is sensible is hard; for example, analysis of referenda results would not be appropriate as referenda are often only held for controversial issues for which a flat prior is unlikely to be appropriate. If, for example, 99% or 1% of a population support a measure, a referendum may not be held, due to easy passing or rejection by a Parliamentary process or not appearing on the political scene.

With the uniform prior, the probability of a dead-heat based on the rest of the population's votes can be calculated:

$$\begin{aligned} p_{tie} &= \int_{q=0}^{1} \binom{K}{\frac{K}{2}} q^{K/2}(1-q)^{K/2}\mathrm{d}q \\ &= \frac{K!}{\left(\frac{K}{2}!\right)^2} \frac{\left(\frac{K}{2}!\right)^2}{(K+1)!} \\ &= \frac{1}{K+1} \propto \frac{1}{N} \end{aligned}$$

where the Beta-integral has been used. This result would suggest that voting weights ought to be allocated in proportion to a country's population (N) to give each individual in the Union an equal voting weight.

3 Voting threshold

A scenario where each country's normalised Banzhaf index (a measure of voting power) closely matches the normalised voting weight is found in [1] to determine a fair voting strategy. This approach gives a threshold of 62% of the voting weight required to pass a motion. It is unfortunate that a threshold greater than 50% makes instigating change harder than maintaining the status quo. The threshold of 62% could be seen in reverse as the threshold required to say "no" – such an approach would make change easier than maintaining the status quo.

However, given the above proposed proportional rather than square-root weighting, a similar threshold was searched for. As the range of voting weights in the proportional model is larger than in the square-root model, the threshold is expected to be further away from [1]'s proposed limit of 0.5 and less perfect. Uniform sampling over percentage points showed that the best threshold is 63% and that the largest relative error for an individual country between voting weight and voting power is 5% (as opposed to 0.2% for the square-root model).

4 Discussion

The approach of [1] relies on the following assumptions:

- The definition of an individual's voting power in terms of probability of holding a casting vote
- The square-root voting weight
- Allocating voting power fairly being the same as the democratic wish

In this paper, an alternative to the square-root voting weight has been proposed but the first and last assumptions have been followed. Further work to study the former and latter assumptions is recommended.

The voting system of [1] does have a pleasing simplicity and elegance but it is not clear that the proposed system is the "correct" way to allocate votes within the European Union.

5 Conclusion

The approach of [1] has been studied. An alternative model based on voting weights being proportional to a country's population has been proposed. It is believed that further work is needed to justify the voting system either proposed by [1] or here.

References

[1] Karol Życzkowski and Wojceich Slomczyński. Voting in the European Union: the square-root system of Penrose and a criticial point. arXiv.org/abs/cond-mat/0405396, May 2004.

[2] Karol Życzkowski, Wojceich Slomczyński, and Tomasz Zastawniak. Physics for fairer voting. *Physics World*, 19(3):35–37, March 2006.

[3] Maths tweak required for EU voting. BBC News Online http://news.bbc.co.uk/1/hi/sci/tech/3804841.stm, July 2004.

[4] The EU constitution is unfair, according to game theorists. The Telegraph, 14 July 2004.

[5] The treaty establishing a constitution for Europe: Part III. In *House of Commons Research Papers*, number 04/75. October 2004.