Topic Modeling: Beyond Bag-of-Words

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Abstract

Some models of textual corpora employ text generation methods involving n-gram statistics, while others use latent topic variables inferred using the “bag-of-words” assumption, in which word order is ignored. Previously, these methods have not been combined. In this work, I explore a hierarchical generative probabilistic model that incorporates both n-gram statistics and latent topic variables by extending a unigram topic model to include properties of a hierarchical Dirichlet bigram language model. The model hyperparameters are inferred using a Gibbs EM algorithm. On two data sets, each of 150 documents, the new model exhibits better predictive accuracy than either a hierarchical Dirichlet bigram language model or a unigram topic model. Additionally, the inferred topics are less dominated by function words than are topics discovered using unigram statistics, potentially making them more meaningful.

1. Introduction

Recently, much attention has been given to generative probabilistic models of textual corpora, designed to identify representations of the data that reduce description length and reveal inter- or intra-document statistical structure. Such models typically fall into one of two categories—those that generate each word on the basis of some number of preceding words or word classes and those that generate words based on latent topic variables inferred from word correlations independent of the order in which the words appear.

Although n-gram language models may use conditioning contexts of arbitrary length, the focus of this paper is restricted to bigram models—i.e., models that predict each word based on the immediately preceding word. To develop a bigram language model, marginal and conditional word counts are determined from a corpus $w$. The marginal count $N_i$ is defined as the number of times that word $i$ has occurred in the corpus, while the conditional count $N_{ij}$ is the number of times word $i$ immediately follows word $j$. Given these counts, the aim of bigram language modeling is to develop predictions of word $w_t$ given word $w_{t-1}$, in any document. Typically this is done by computing estimators of both the marginal probability of word $i$ and the conditional probability of word $i$ following word $j$, such as $f_i = N_i / N$ and $f_{ij} = N_{ij} / N_j$, where $N$ is the number of words in the corpus. $f_{ij}$ is then smoothed by the marginal frequency estimator $f_i$ to give the predictive probability of $i$ given $j$:

$$P(w_t = i | w_{t-1} = j) = \lambda f_i + (1 - \lambda) f_{ij},$$

(1)

The parameter $\lambda$ may be fixed, or determined from the data using techniques such as cross-validation (Jelinek & Mercer, 1980). This procedure works well in practice, despite its somewhat ad hoc nature.

The hierarchical Dirichlet language model (MacKay & Peto, 1995) is a bigram model that is entirely driven by principles of Bayesian inference. This model has a similar predictive distribution to models based on equation (1), with one key difference: the bigram statistics $f_{ij}$ in MacKay and Peto’s model are not smoothed with marginal statistics $f_i$, but are smoothed with a quantity related to the number of different contexts in which each word has occurred.

Latent Dirichlet allocation (Blei et al., 2003) provides an alternative approach to modeling textual corpora. Documents are modeled as finite mixtures over an underlying set of latent topics inferred from correlations between words, independent of word order.

This bag-of-words assumption makes sense from a point of view of computational efficiency, but is unrealistic. In many language modeling applications, such as text compression, speech recognition, and predictive text entry, word order is extremely important. Furthermore, it is likely that word order can assist in topic
inference. The phrases “the department chair couches offers” and “the chair department offers couches” have the same unigram statistics, but are about quite different topics. When deciding which topic generated the word “chair” in the first sentence, knowing that it was immediately preceded by the word “department” makes it much more likely to have been generated by a topic that assigns high probability to words related to university administration.

In practice, the topics inferred using latent Dirichlet allocation are heavily dominated by function words, such as “in”, “that”, “of” and “for”, unless these words are removed from corpora prior to topic inference. While removing these may be appropriate for tasks where word order does not play a significant role, such as information retrieval, it is not appropriate for many language modeling applications, where both function and content words must be accurately predicted.

In this paper, I present a hierarchical Bayesian model that integrates bigram-based and topic-based approaches to document modeling. This model moves beyond the bag-of-words assumption found in latent Dirichlet allocation by introducing properties of MacKay and Peto’s hierarchical Dirichlet language model. In addition to exhibiting better predictive performance than either MacKay and Peto’s language model or latent Dirichlet allocation, the topics inferred using the new model are typically less dominated by function words than are topics inferred from the same corpora using latent Dirichlet allocation.

2. Background

I begin with brief descriptions of MacKay and Peto’s hierarchical Dirichlet language model and Blei et al.’s latent Dirichlet allocation.

2.1. Hierarchical Dirichlet language model

Bigram language models are specified by a conditional distribution \( P(w_t = i | w_{t-1} = j) \), described by \( W(W - 1) \) free parameters, where \( W \) is the number of words in the vocabulary. These parameters are denoted by the matrix \( \Phi \), with \( P(w_t = i | w_{t-1} = j) \equiv \phi_{ij} \). \( \Phi \) may be thought of as a transition probability matrix, in which the \( j \)th row, the probability vector for transitions from word \( j \), is denoted by the vector \( \phi_j \).

Given a corpus \( w \), the likelihood function is

\[
P(w | \Phi) = \prod_i \prod_j N_{ij} \phi_{ij}^N_{ij},
\]

where \( N_{ij} \) is the number of times that word \( i \) immediately follows word \( j \) in the corpus.

MacKay and Peto (1995) extend this basic framework by placing a Dirichlet prior over \( \Phi \):

\[
P(\Phi | \beta m) = \prod_j \text{Dirichlet}(\phi_j | \beta m),
\]

where \( \beta > 0 \) and \( m \) is a measure satisfying \( \sum_i m_i = 1 \). Combining equations (2) and (3), and integrating over \( \Phi \), yields the probability of the corpus given the hyperparameters \( \beta m \), also known as the “evidence”:

\[
P(w | \beta m) = \prod_j \frac{\prod_i \Gamma(N_{ij} + \beta m_i)}{\Gamma(N_j + \beta)} \frac{\Gamma(\beta)}{\prod_i \Gamma(\beta m_i)}. \tag{4}
\]

It is also easy to obtain a predictive distribution for each context \( j \) given the hyperparameters \( \beta m \):

\[
P(i | j, w, \beta m) = \frac{N_{ij} + \beta m_i}{N_j + \beta}. \tag{5}
\]

To make the relationship to equation (1) explicit, \( P(i | j, w, \beta m) \) may be rewritten as

\[
P(i | j, w, \beta m) = \lambda_j m_i + (1 - \lambda_j) f_{ij}, \tag{6}
\]

where \( f_{ij} = N_{ij} / N_j \) and

\[
\lambda_j = \frac{\beta}{N_j + \beta}. \tag{7}
\]

The hyperparameter \( m_i \) is now taking the role of the marginal statistic \( f_i \) in equation (1).

MacKay and Peto assume an uninformative prior over the hyperparameters \( \beta m \), yielding a minimal data-driven Bayesian model in which the optimal hyperparameters, \( \beta m \)\textsuperscript{MP}, may be determined by maximizing the evidence. MacKay and Peto show that each element of \( m \)\textsuperscript{MP}, when estimated using this “empirical Bayes” procedure, is related to the number of contexts in which the corresponding word has appeared.

2.2. Latent Dirichlet allocation

Latent Dirichlet allocation (Blei et al., 2003) represents documents as random mixtures over latent topics, where each topic is characterized by a distribution over words. Each word \( w_t \) in a corpus \( w \) is assumed to have been generated by a latent topic \( z_t \), drawn from a document-specific distribution over \( T \) topics.

Word generation is defined by a conditional distribution \( P(w_t = i | z_t = k) \), described by \( T(W - 1) \) free parameters, where \( T \) is the number of topics and \( W \) is the size of the vocabulary. These parameters are denoted by \( \Phi \), with \( P(w_t = i | z_t = k) \equiv \phi_{ijk} \). \( \Phi \) may be
thought of as an emission probability matrix, in which the \( k^{th} \) row, the distribution over words for topic \( k \), is denoted by \( \phi_k \). Similarly, topic generation is characterized by a conditional distribution \( P(z_t = k | d_t = d) \), described by \( D(T - 1) \) free parameters, where \( D \) is the number of documents in the corpus. These parameters form a matrix \( \Theta \), with \( P(z_t = k | d_t = d) = \theta_{kd} \). The \( d^{th} \) row of this matrix is the distribution over topics for document \( d \), denoted by \( \theta_d \).

The joint probability of a corpus \( w \) and a set of corresponding latent topics \( z \) is

\[
P(w, z | \Phi, \Theta) = \prod_i \prod_k \prod_d \phi_{ik}^{N_{ik}} \theta_{kd}^{N_{kd}}, \tag{8}
\]

where \( N_{ik} \) is the number of times that word \( i \) has been generated by topic \( k \), and \( N_{kd} \) is the number of times topic \( k \) has been used in document \( d \).

Blei et al. place a Dirichlet prior over \( \Phi \),

\[
P(\Phi|\beta m) = \prod_k \text{Dirichlet}(\phi_k | \beta m), \tag{9}
\]

and another over \( \Theta \),

\[
P(\Theta|\alpha n) = \prod_d \text{Dirichlet}(\theta_d | \alpha n). \tag{10}
\]

Combining these priors with equation (8) and integrating over \( \Phi \) and \( \Theta \) gives the evidence for hyperparameters \( \alpha n \) and \( \beta m \), which is also the probability of the corpus given the hyperparameters:

\[
P(w|\alpha n, \beta m) = \sum_z \left( \prod_k \frac{\prod_i \Gamma(N_{ik} + \beta m_i) \Gamma(\beta)}{\Gamma(N_k + \beta) \prod_i \Gamma(\beta m_i)} \right) \left( \prod_d \frac{\prod_i \Gamma(N_{kd} + \alpha n_k) \Gamma(\alpha)}{\Gamma(N_d + \alpha) \prod_i \Gamma(\alpha n_k)} \right). \tag{11}
\]

\( N_k \) is the total number of times topic \( k \) occurs in \( z \), while \( N_d \) is the number of words in document \( d \).

The sum over \( z \) cannot be computed directly because it does not factorize and involves \( T^N \) terms, where \( N \) is the total number of words in the corpus. However, it may be approximated using Markov chain Monte Carlo (Griffiths & Steyvers, 2004).

Given a corpus \( w \), a set of latent topics \( z \), and optimal hyperparameters \( \alpha n^{\text{MP}} \) and \( \beta m^{\text{MP}} \), approximate predictive distributions for each topic \( k \) and document \( d \) are given by the following pair of equations:

\[
P(i | k, w, z, \beta m^{\text{MP}}) = \frac{N_{ijk} + \beta m_i^{\text{MP}}}{N_k + \beta^{\text{MP}}} \tag{12}
\]

\[
P(k | d, w, z, \alpha n^{\text{MP}}) = \frac{N_{k|d} + \alpha n_k^{\text{MP}}}{N_d + \alpha^{\text{MP}}} \tag{13}
\]

Note the similarity of these equations to equation (6).

3. Bigram topic model

This section introduces a model that extends latent Dirichlet allocation by incorporating a notion of word order, similar to that employed by MacKay and Peto’s hierarchical Dirichlet language model. Each topic is now represented by a set of \( W \) distributions.

Word generation is defined by a conditional distribution \( P(w_t = i | w_{t-1} = j, z_t = k) \), described by \( WT(W - 1) \) free parameters. As before, these parameters form a matrix \( \Phi \), this time with \( WT \) rows. Each row is a distribution over words for a particular context \( j, k \), denoted by \( \phi_{j,k} \). Each topic \( k \) is now characterized by the \( W \) distributions specific to that topic. Topic generation is the same as in latent Dirichlet allocation: topics are drawn from the conditional distribution \( P(z_t = k | d_t = d) \), described by \( D(T - 1) \) free parameters, which form a matrix \( \Theta \).

The joint probability of a corpus \( w \) and a single set of latent topic assignments \( z \) is

\[
P(w, z | \Phi, \Theta) = \prod_i \prod_j \prod_k \prod_d \phi_{ij,k}^{N_{ij,k}} \theta_{kd}^{N_{kd}}, \tag{14}
\]

where \( N_{ij,k} \) is the number of times word \( i \) has been generated by topic \( k \) when preceded by word \( j \). As in latent Dirichlet allocation, \( N_{kd} \) is the number of times topic \( k \) has been used in document \( d \).

The prior over \( \Theta \) is chosen to be the same as that used in latent Dirichlet allocation:

\[
P(\Theta | \alpha n) = \prod_d \text{Dirichlet}(\theta_d | \alpha n). \tag{15}
\]

However, the additional conditioning context \( j \) in the distribution that defines word generation affords greater flexibility in choosing a hierarchical prior for \( \Phi \) than in either latent Dirichlet allocation or the hierarchical Dirichlet language model. The priors over \( \Phi \) used in both MacKay and Peto’s language model and Blei et al.’s latent Dirichlet allocation are “coupled” priors: learning the probability vector for a single context, \( \phi_j \), the case of MacKay and Peto’s model and \( \phi_k \) in Blei et al.’s, gives information about the probability vectors in other contexts, \( j' \) and \( k' \) respectively. This dependence comes from the hyperparameter vector \( \beta m \), shared, in the case of the hierarchical Dirichlet language model, between all possible previous word contexts \( j \) and, in the case of latent Dirichlet allocation, between all possible topics \( k \). Since word generation is conditioned upon both \( j \) and \( k \) in the new model presented in this paper, there is more than
one way in which hyperparameters for the prior over \( \Phi \) might be shared in this model.

**Prior 1:** Most simply, a single hyperparameter vector \( \beta m \) may be shared between all \( j,k \) contexts:

\[
P(\Phi|\beta m) = \prod_j \prod_k \text{Dirichlet}(\phi_{j,k}|\beta m). \tag{16}
\]

Here, knowledge about the probability vector for one \( \phi_{j,k} \) will give information about the probability vectors \( \phi_{j',k} \) for all other \( j',k' \) contexts.

**Prior 2:** Alternatively there may be \( T \) hyperparameter vectors—one for each topic \( k \):

\[
P(\Phi|\beta_k m_k) = \prod_j \prod_k \text{Dirichlet}(\phi_{j,k}|\beta_k m_k). \tag{17}
\]

Information is now shared between only those probability vectors with topic context \( k \). Intuitively, this is appealing. Learning about the distribution over words for a single context \( j,k \) yields information about the distributions over words for other contexts \( j',k \) that share this topic, but not about distributions with other topic contexts. In other words, this prior encapsulates the notion of similarity between distributions over words for a given topic context.

The evidence, or probability of a corpus \( w \) given the hyperparameters, is either (prior 1)

\[
P(w|\alpha n, \beta m) = \\
\sum_z \left( \prod_j \prod_k \frac{\Gamma(N_{ij,k} + \beta m_i)}{\Gamma(N_{j,k} + \beta)} \frac{\Gamma(\beta)}{\prod_i \Gamma(\beta m_i)} \right) \\
\prod_d \frac{\Gamma(N_{k|d} + \alpha n_k)}{\Gamma(N_d + \alpha)} \frac{\Gamma(\alpha)}{\prod_k \Gamma(\alpha n_k)} \tag{18}
\]

or (prior 2)

\[
P(w|\alpha n, \{\beta_k m_k\}) = \\
\sum_z \left( \prod_j \prod_k \frac{\Gamma(N_{ij,k} + \beta m_{i|k})}{\Gamma(N_{j,k} + \beta_k)} \frac{\Gamma(\beta)}{\prod_i \Gamma(\beta m_{i|k})} \right) \\
\prod_d \frac{\Gamma(N_{k|d} + \alpha n_k)}{\Gamma(N_d + \alpha)} \frac{\Gamma(\alpha)}{\prod_k \Gamma(\alpha n_k)} \tag{19}
\]

As in latent Dirichlet allocation, the sum over \( z \) is intractable, but may be approximated using MCMC.

For a single set of latent topics \( z \), and optimal hyperparameters \( |\beta m|^{\text{MP}} \) or \( |\beta_k m_k|^{\text{MP}} \), the approximate predictive distribution over words given previous word \( j \) and current topic \( k \) is either (prior 1)

\[
P(i|j, k, w, z, |\beta m|^{\text{MP}}) = \frac{N_{ij,k} + |\beta m|^{\text{MP}}}{N_{j,k} + |\beta m|^{\text{MP}}} \tag{20}
\]

or (prior 2)

\[
P(i|j, k, w, z, |\{\beta_k m_k|^{\text{MP}}\}) = \frac{N_{ij,k} + |\{\beta_k m_k|^{\text{MP}}\}}{N_{j,k} + |\beta m|^{\text{MP}}} \tag{21}
\]

In equation (20), the statistic \( N_{ij,k}/N_{j,k} \) is always smoothed by the quantity \( m_i \), regardless of the conditioning context, \( j,k \). Meanwhile, in equation (21), \( N_{ij,k}/N_{j,k} \) is smoothed by \( m_{i|k} \), which is will vary depending on the conditioning topic \( k \).

Given \( |\alpha n|^{\text{MP}} \), the approximate predictive distribution over topics for document \( d \) is

\[
P(k|d, w, z, |\alpha n|^{\text{MP}}) = \frac{N_{k|d} + |\alpha n|^{\text{MP}}}{N_d + |\alpha n|^{\text{MP}}} \tag{22}
\]

**4. Inference of hyperparameters**

Previous sampling-based treatments of latent Dirichlet allocation have not included any method for optimizing hyperparameters. However, the method described in this section may be applied to both latent Dirichlet allocation and the model presented in this paper.

Given an uninformative prior over \( \alpha n \) and \( \beta m \) or \( \{\beta_k m_k\} \), the optimal hyperparameters, \( |\alpha n|^{\text{MP}} \) and \( |\beta m|^{\text{MP}} \) or \( |\{\beta_k m_k|^{\text{MP}}\} \), may be found by maximizing the evidence, given in equation (18) or (19).

The evidence contains latent variables \( z \) and must therefore be maximized with respect to the hyperparameters using an expectation-maximization (EM) algorithm. Unfortunately, the expectation with respect to the distribution over the latent variables involves a sum over \( T^N \) terms, where \( N \) is the number of words in the entire corpus. However, this sum may be approximated using a Markov chain Monte Carlo algorithm, such as Gibbs sampling, resulting in a Gibbs EM algorithm (Andrieu et al., 2003).

**5. Experiments**

To evaluate the new model, both variants were compared with latent Dirichlet allocation and MacKay and Peto’s hierarchical Dirichlet language model. The topic models were trained identically: the same Gibbs EM algorithm was used for both the new model (with either prior) and latent Dirichlet allocation. The hyperparameters of the hierarchical Dirichlet language
model were inferred using the same fixed-point iteration used in the M-step of the Gibbs EM algorithm.

Language models are typically evaluated by computing the information rate of unseen test data, measured in bits per word: the better the predictive performance, the fewer the bits per word. Information rate is a direct measure of text compressibility. Given corpora $w$ and $w_{test}$, information rate is defined as

$$R = -\frac{\log_2 P(w_{test} | w)}{N_{test}},$$

(23)

where $N_{test}$ is the number of words in the test corpus. The information rate may be computed directly for the hierarchical Dirichlet language model. For the topic models, computing $P(w_{test} | w)$ requires summing over $z$ and $z_{test}$. As mentioned before, this is intractable. Instead, the information rate may be computed using a single set of topics $z$ for the training data, in this case obtained by running a Gibbs sampler for 20000 iterations after the hyperparameters have been inferred. Given $z$, multiple sets of topics for the test data $\{z_{test}^{(s)}\}_{s=1}^S$ may be obtained using the predictive distributions. Given hyperparameters $U$, $P(w_{test} | w)$ may be approximated by taking the harmonic mean of $\{P(w_{test} | z_{test}^{(s)} , w, z, U)\}_{s=1}^S$ (Kass & Raftery, 1995).

5.1. Corpora

The models were compared using two data sets. The first was constructed by drawing 150 abstracts (documents) at random from the Psychological Review Abstracts data provided by Griffiths and Steyvers (2005). A subset of 100 documents were used to infer the hyperparameters, while the remaining 50 were used for evaluating the models. The second corpus consisted of 150 newsgroup postings, drawn at random from the 20 Newsgroups data (Rennie, 2005). Again, 100 documents were used for inference, while 50 were retained for evaluating predictive accuracy.

5.2. Results

The experiments involving latent Dirichlet allocation and the new model were run with 1 to 120 topics. The information rates of the test data are shown in figure 1. On both corpora, latent Dirichlet allocation and the hierarchical Dirichlet language model achieve similar performance. With prior 1, the new model improves upon this by between 0.5 and 1 bits per word. However, with prior 2, it achieves an information rate reduction of between 1 and 2 bits per word. For latent Dirichlet allocation, the information rate is reduced most by the first 20 topics. The new model uses a larger number of topics and exhibits a greater information rate reduction as more topics are added. In latent Dirichlet allocation, the latent topic for a given word is inferred using the identity of the word, the number of times the word has previously been assumed to be generated by each topic, and the number of times each topic has been used in the current document. In the new model, the previous word is also taken into account. This additional information means words that were considered to be generated by the same topic in latent Dirichlet allocation, may now be assumed to have been generated by different topics, depending on the contexts in which they are seen. Consequently, the new model tends to use a greater number of topics.

In addition to comparing predictive accuracy, it is instructive to look at the inferred topics. Table 1 shows the words most frequently assigned to a selection of topics extracted from the 20 Newsgroups training data by each of the models. The “unseen” token was omitted. The topics inferred using latent Dirichlet allocation contain many function words, such as “the”,
“in” and “to”. In contrast, all but one of the topics inferred by the new model, especially with prior 2, typically contain fewer function words. Instead, these are largely collected into the single remaining topic, shown in the last column of rows 2 and 3 in Table 1. This effect is similar, though less pronounced, to that achieved by Griffiths et al.’s composite model (2004), in which function words are handled by a hidden Markov model, while content words are handled by latent Dirichlet allocation.

### 6. Conclusions

Creating a single model that integrates bigram-based and topic-based approaches to document modeling has several benefits. Firstly, the predictive accuracy of the new model, especially when using prior 2, is significantly better than that of either latent Dirichlet allocation or the hierarchical Dirichlet language model. Secondly, the model automatically infers a separate topic for function words, meaning that the other topics are less dominated by these words.

### References


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### Table 1

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<tr>
<th>TOPIC</th>
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All three sets of topics were taken from models with 90 topics.