# (4096,3249) Gallager Codes compared with Tanner Product Codes

David J. C. MacKay

Department of Physics, University of Cambridge
Cavendish Laboratory, Madingley Road,
Cambridge, CB3 0HE, United Kingdom.
mackay@mrao.cam.ac.uk

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**Abstract**

We present three Gallager codes with blocklength $N = 4096$, and rate $R = 3249/4096 = 0.793$ and compare them with the corresponding Tanner Product Code (TPC). [Notes written Tue 13/2/01, Original work done 11/12/99.]

## "Turbo product code" comparison

I prefer to call "Turbo product codes" "Tanner product codes" since Tanner (1981) invented them long before Turbo codes were invented.

### BACKGROUND

The TPC I have compared with has rate $R = 0.793$ and blocklength $N = 4096$. My approach was to test three codes: two regular Gallager codes (with $t = 3$ and $t = 4$) (Gallager, 1963; MacKay and Neal, 1996; MacKay, 1999), and one irregular code (Luby *et al.*, 2001; Chung *et al.*, 2001) (actually, I made about 6 trial irregular codes of which this was the best so far). I put little effort into optimization of the codes. The experiments were intended to be quick, to get a ball-park answer to the question "Are TPC's easy to match?"

Theoretically, by the way, we know that TPC's are not great asymptotically because the distance of a product of two codes with distance $d_1$ and $d_2$ is at best $d_1 d_2$, so the fractional distance $(d/N)$ goes down with productification.

### RESULTS

The performance curves for all three codes and the TPC are essentially equivalent down to 1e-6. (The differences are less than 0.25dB.)

As usual, the performance curves in the literature show bit error probability and do not distinguish detected and undetected errors – a practice of which I am critical, since I think the distinction is important. The regular Gallager codes (t=3, t=4) made no undetected errors. Assuming that TPCs make undetected errors (which I expect is the case, because they are product codes and have distance only 9), this feature of LDPCs could be a practical advantage of Gallager codes in some applications. If someone could supply a graph of the block error rate of the TPC then I could include a second figure comparing the block error rates.

The irregular code has a slightly better high-signal-to-noise-ratio behaviour than the other codes, and it also has an error floor with undetected errors. I think that by optimizing the irregular code, this error floor could be removed; but that would take more research effort.

### MORE DETAILS

The first and second codes are regular codes over GF(2) with column weights 3 and 4 respectively. The third is an irregular code over GF(2) with a profile of column weights

and row weights that were found with the aid of S–Y. Chung's online profile optimizer `http://truth.mit.edu/~sychung/gaopt.html` (Chung *et al.*, 2001) moderated by a dose of human experience.
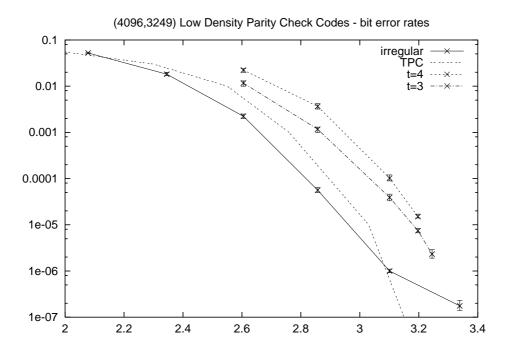


Figure 1. Performance of Gallager codes and TPC: Bit error rate as a function of $E_b/N_0$.

## References

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