

Two Small Gallager Codes

David J.C. MacKay and Matthew C. Davey
Department of Physics, University of Cambridge
Cavendish Laboratory, Madingley Road,
Cambridge, CB3 0HE, United Kingdom.
`mackay|mcdavey@mrao.cam.ac.uk`

Abstract

We present a pair of Gallager codes with rate $R = 1/3$ and transmitted blocklength $N = 1920$ as candidates for the proposed international standard for cellular telephones.

A regular Gallager code (Gallager, 1962) has a parity check matrix with uniform column weight j and uniform row weight k , both of which are very small compared to the blocklength. If the code has transmitted blocklength N and rate R then the parity check matrix \mathbf{H} has N columns and M rows, where $M \geq N(1 - R)$. [Normally parity check matrices have $M = N(1 - R)$, but the matrices we construct may have a few redundant rows so that their rate could be a little higher than $1 - M/N$.]

It is easy to make good Gallager codes. We have found that for just about any blocklength N and rate R , a randomly chosen Gallager code with $j \simeq 3$ gives performance (in terms of word error probability on a Gaussian channel with signal to noise ratio E_b/N_0) that is within a fraction of a decibel of the best known codes (MacKay and Neal, 1996). Furthermore, with a little effort, irregular Gallager codes can be found which equal, or even exceed, the performance of what were the best known codes (Davey and MacKay, 1998; Urbanke *et al.*, 1999).

We show in figure 1 the performance of two rate $1/3$ codes with transmitted blocklength $N = 1920$ and source blocklength $K = 640$ that we constructed and tested within the space of one week. The first code is a nearly-regular code over $\text{GF}(2)$ with column weights 3 and 2 and row weight 4 [Construction 2A from (MacKay and Neal, 1996)]. The second is an irregular code over $\text{GF}(4)$ with a profile of column weights and row weights that was found by a brief Monte Carlo search. The irregular code was constructed according to the profile given in figure 3, using the Poisson construction method described in (MacKay *et al.*, 1999). The irregular code — which we expect could be further improved — is less than 0.2dB from the line showing the performance of the CCSDS turbo codes on the JPL code imperfectness web-page (JPL, 1999).

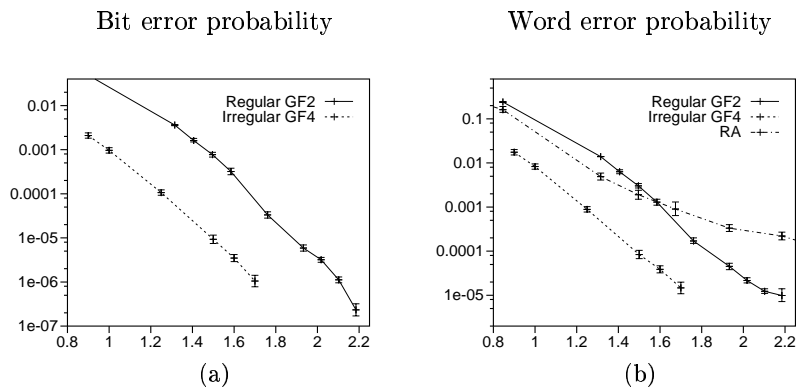


Figure 1. (a,b) Performance of Gallager codes with $N = 1920$, $R = 1/3$, as a function of E_b/N_0 . In (b) we also show the performance of a repeat-accumulate code with $N = 3000$.

Gallager codes have advantages that these comparisons do not make evident. First, whereas turbo codes sometimes make undetected errors, Gallager codes are found to make only detected errors — all incorrectly decoded blocks are flagged by the decoder. Second, their decoding complexity is low — lower than that of turbo codes. The costs per bit per iteration are about 4 additions and 18 multiplies for the binary code, and 14 additions and 31 multiplies for the $GF(4)$ code. The expected number of iterations depends on the noise level, as we now describe.

Decoding times follow power laws

We have previously noted (MacKay, 1999b; MacKay *et al.*, 1999) that the probability distribution of the decoding time τ of Gallager codes and turbo-like codes appears to follow a power law, $P(\tau) \propto \tau^{-p}$, with the exponent p depending on the noise level and the code. Figure 2 shows the histograms of decoding times at four signal-to-noise ratios for the binary Gallager code shown in figure 1.

The parity check matrix of the binary code, 1920.1280.3.303, can be found in the online archive (MacKay, 1999a). We expect that these codes could be further improved by a careful optimization of their profiles.

Acknowledgements

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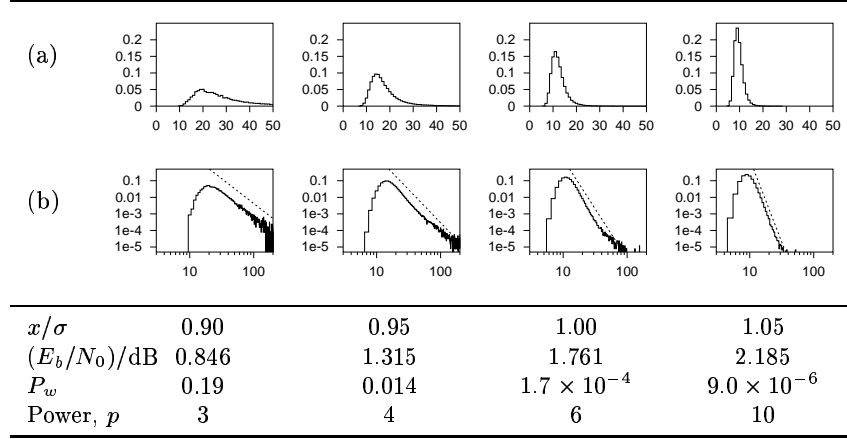


Figure 2. Histograms showing the frequency distribution of decoding times for the binary Gallager code from figure 1: (a) linear plot; (b) log-log plot. The graphs show the number of iterations taken to reach a valid decoding; the value of P_w gives the frequency with which no valid decoding was reached after 1000 iterations. The power p which gives a good fit of the power law distribution $P(\tau) \propto \tau^{-p}$ (for large τ) is also shown.

Columns		Rows	
2	449	5	635
3	389	6	5
4	72		
5	1		
11	2		
13	4		
19	39		

Figure 3. The profile of the irregular code over $GF(4)$, whose blocklength was 960 symbols.

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