Darwin Magazine Puzzle

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Figure 1. A big rectangle filled with little rectangles. All the red rectangles have integer height. (For example, the bottom-left red rectangle has height 8.) All the green rectangles have integer width.

A number of small rectangles perfectly tile a big rectangle. If the small rectangles all have the property that at least one of their sides is an integer, can you show that the big rectangle also has at least one integer side? That is, either the width of the big rectangle is an integer, or its height is an integer, or both.