

# Time Warp Invariance by Recoding Time Delays into Time Delays

David J.C. MacKay  
University of Cambridge

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## Abstract

A little investigation of how easy it is to get time warp invariance using your trick of coupling a decaying response  $g(t)$  to a time advance associated with a subthreshold oscillation  $s(t)$ .

This work is an exploration of the original ideas of John Hopfield. It has not been submitted for publication.

## 1 Parameterization

Let a sensory cell respond to the detection of its event ‘A’ with a decaying response:

$$g(t) = 0.5(\exp(-t/\lambda_1) + \exp(-t/\lambda_2)). \quad (1)$$

Notice that here I have effectively chosen one free parameter  $\lambda_2/\lambda_1$ , since the units of time are arbitrary. Figure 1 shows the example function

$$g(t) = 0.5(\exp(-t) + \exp(-t/10)). \quad (2)$$

Let this sensory cell be attached to a spiking cell, which does its communication for it, and has a subthreshold oscillation:

$$s(\tau) = \cos(\tau + a \sin(\tau - b) + \sin(b)). \quad (3)$$

This oscillation is imagined to be happening on a much faster time-scale than the dynamics of the outside world and of  $g(t)$ . This is a two-parameter oscillation, with  $a$  ( $|a| < 1$ ) controlling the magnitude of non-sinusoidality and  $b$  controlling the phase in the cycle of the non-sinusoidality. The term  $\sin(b)$  is included for convenience simply to fix the locations of the maxima at  $\tau = 2n\pi$ . Figure 2 shows the example function

$$s(\tau) = \cos(\tau + 0.8 \sin(\tau - 2.6) + \sin(2.6)). \quad (4)$$

The threshold of the spiking cell is assumed to be 1.0, so that its oscillations bring it exactly to threshold if it is unexcited (*i.e.*, if  $g(t) = 0$ ).

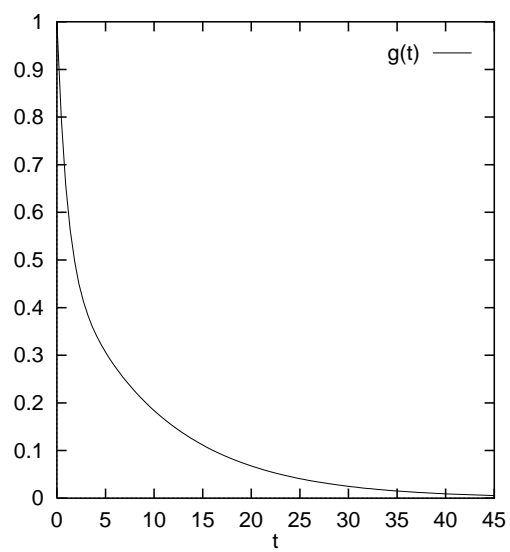


Figure 1:  $g(t)$

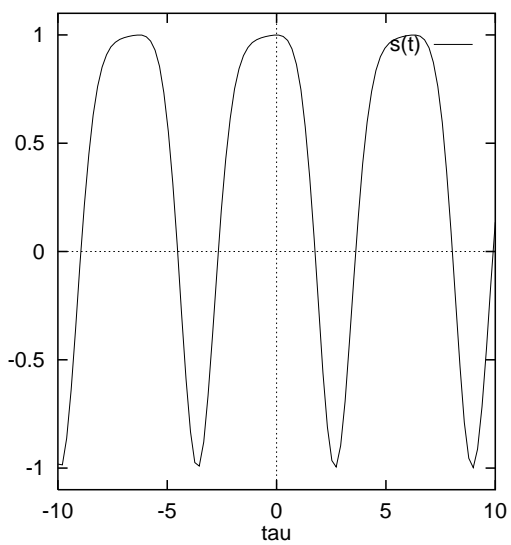


Figure 2:  $s(\tau)$

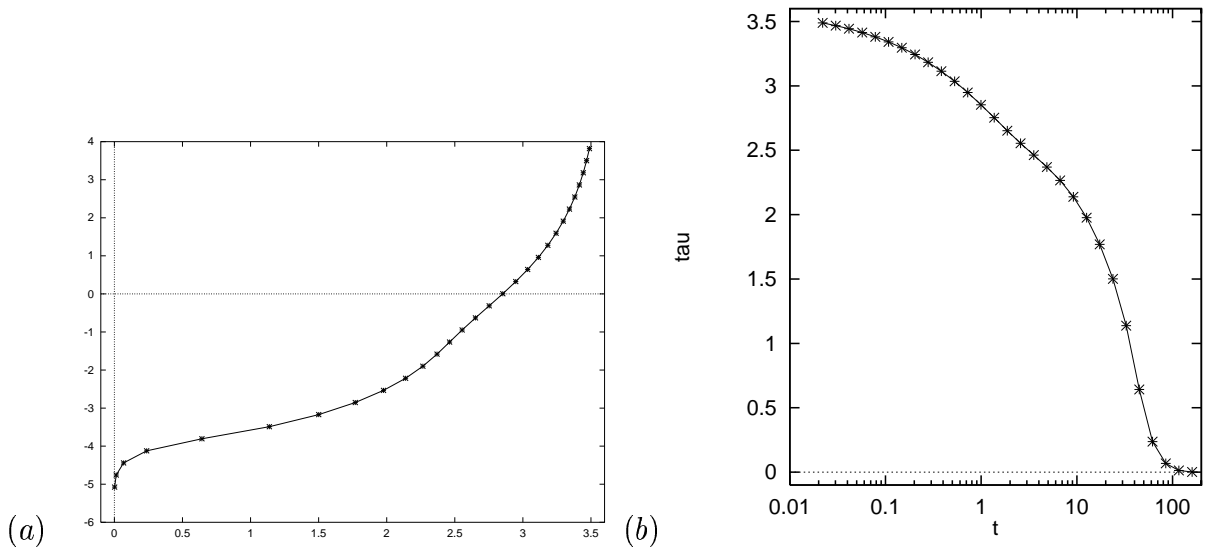


Figure 3: (a)  $-\log_e(t)$  versus the time advance  $\tau$ .  $t = 0.022, 0.030, 0.041, 0.057 \dots 61, 85, 116, 160$ . (b) Same graph, showing  $\tau$  versus  $t$  on log scale.

We now assume that the potential of the oscillating cell is incremented by  $2g(t)$  and examine the magnitude of the time advance  $\tau(t)$  as a function of  $t$ .<sup>1</sup> To determine the time-advance  $\tau$  I solved this equation numerically:

$$s(\tau(t)) + 2g(t) = 1. \quad (5)$$

‘Time-warp invariance’ is achieved if  $\tau(t)$  is proportional to  $\log(t)$ .

Figure 3 shows the time advance  $\tau(t)$  on the horizontal axis and  $-\log_e(t)$  on the vertical axis, with  $t$  varying over more than **three** orders of magnitude from  $t = 0.022, 0.030, 0.041, 0.057 \dots$  to  $t = 61, 85, 116, 160$ . The largest time advances are associated with the smallest values of  $t$ . Notice that for  $-\log_e(t)$  from  $-4.5$  to  $3.0$  the curve is not far from a straight line.

The values of the three parameters used here were set by hand without making any extensive search. I just looked for an oscillation which had the right sort of shape, then set  $\lambda_2/\lambda_1$  to 10 on the grounds that that would be the best way to get something that worked over one order of magnitude. Perturbations to these parameters do not seem to improve the picture very much.

## 2 Conclusion

With only three tweakable parameters we can get something with a great deal of time-warp invariance. By adding one more parameter (for example a second parameter in  $g$ ) and doing an optimization, we could probably get a very good fit to  $-\log_e(t)$ .

<sup>1</sup>The factor of 2 is there so that the dynamic range of the oscillation matches that of  $g(t)$ .