Big Things that Squeeze Round Corners

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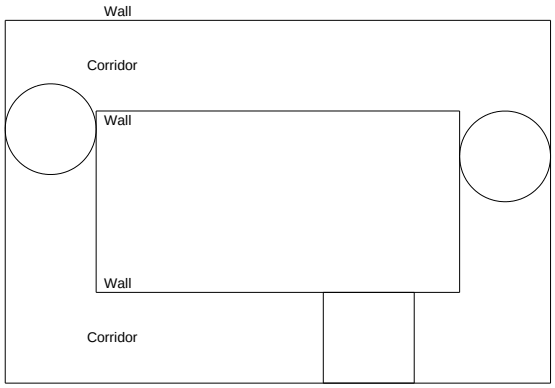
No man is an island, entire of itself. But almost every man is a furniture-mover from time to time. And woman too of course.

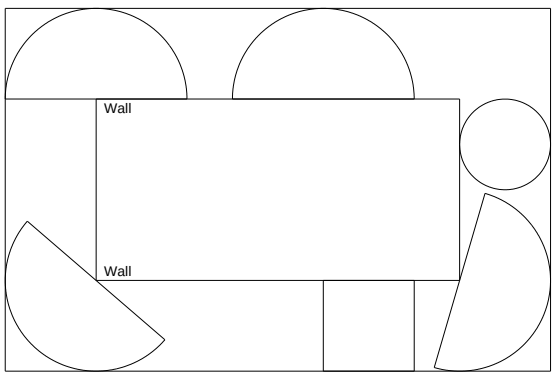
As furniture movers, we always ask ourselves: 'will that bed/desk/wardrobe go up that staircase?' After grunting and shoving for a few minutes, we wonder 'hmm, would it go round the corner if I sawed that leg off?' After sawing off all four legs and hacking away the banisters with an axe, we give up and decide to move into a different house.

In this article, I explore the question, 'what is the largest piece of furniture that can be squeezed round a corner?'

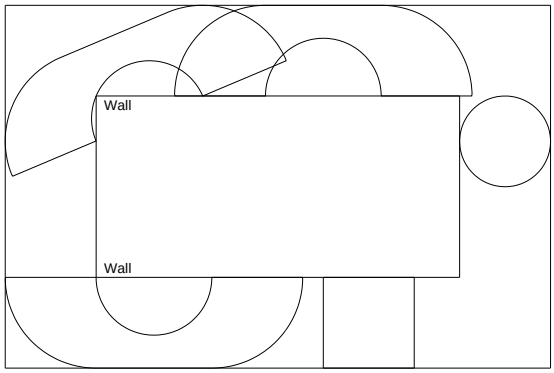
Even in a two-dimensional flatland, I don't know a mathematical technique for tackling the problem. The task is to make a rigid object with the largest possible area that can be taken round a corner in a flatland corridor of width 1.



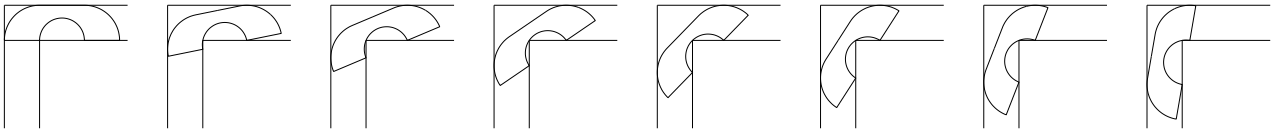
The figure shows a corridor of width 1, and two simple objects, a circle and a square of areas $\pi/4 \simeq 0.79$ and 1, respectively, both of which can slide along the corridor and go round the corners.



The third figure shows a semicircle of radius 1 and area $\pi/2 \simeq 1.57$.



The second figure shows a telephone-shaped object, whose area is $\pi/2 + 2/\pi \simeq = 2.21$. It is built from two quarter-circles of radius 1 and a rectangle of size $1 \times (2/\pi)$, with a semi-circle chomped out of it. This is the largest object I have found that can go round the corner.



Is it the largest? Perhaps this field of furniture design is still an active research area.

Reading the literature...

(Added Thu 4/9/03.)

I usually read the literature a few years after I started working on a problem. The telephone-shaped object above is called the Hammersley sofa, and it's not the largest possible piece of furniture. The 'Gerver sofa' looks like the Hammersley sofa, but has a more complicated and smoother perimeter. Whereas the Hammersley sofa has area 2.2074 square units, the area of the Gerver sofa is 2.2195, an increase of 0.5%. It remains an open problem to prove whether the Gerver sofa is the largest shape that will fit through the corridor. Internet references:
`http://www.studyworksonline.com/cda/content/article/0,,EXP1755_NAV2-95_SAR1766,00.shtml`
`http://www.mathcad.com/library/constants/sofa.htm`

Another puzzle

Here's another puzzle relating to furniture-moving.

The league for the confounding of furniture-movers devotes its energy to making pieces of two-dimensional furniture that cannot be moved down unit-width corridors – not even down straight ones. And they are interested in *minimizing* the amount of materials that goes into their objects.

For example, an L-shape (right-angled) piece of stiff wire of length slightly more than $2\sqrt{2}$ cannot be moved down the corridor. How much better than this can you do? If you are allowed to weld pieces of stiff wire to each other, I think the optimal solution is easy to find. But purists in the league like to make their unmoveable objects out of a single bent piece of wire, with the shortest possible length.

References for the Moving Sofa constant

H. T. Croft, K. J. Falconer and R. K. Guy, Unsolved Problems in Geometry, Springer-Verlag, 1991, section G5; MR 95k:5200.

J. L. Gerver, On moving a sofa around a corner, Geometriae Dedicata 42 (1992) 267-283; MR 93d:51040.

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