Decoding Times of Irregular Gallager Codes

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Abstract

This paper is an appendix to *'Comparison of Constructions of Irregular Gallager Codes'* by David J. C. MacKay, Simon T. Wilson and Matthew C. Davey, which was published in the proceedings of the 1998 Allerton Conference on Communication, Control, and Computing and submitted to IEEE Transactions on Communications 30 July 1998. That paper compares alternative methods for constructing irregular Gallager codes.

This paper reports the decoding times of the codes studied in that paper. The decoding time differs very little between irregular codes and regular codes.

I prepared this draft for Dan Spielman. Dan, should I write this up for publication somewhere? Perhaps in the *IEEE Transactions on Sparse Graph Codes*? Oh, that doesn't exist yet does it.

1 Introduction

It is natural to speculate that irregular Gallager codes might take longer to decode than regular Gallager codes. This paper presents the facts, for the case of a small collection of well–studied codes.

The decoding time can be described by the number of iterations of the sum–product algorithm required. One iteration is a horizontal step followed by a vertical step. Given one code and a set of channel conditions the decoding time varies randomly from trial to trial. Figure 1 shows two histograms of the number of iterations to get a valid decoding under two channel conditions. (Cases where no valid decoding was found are not included in the histograms.) When the signal to noise ratio is higher, fewer iterations are required. In the remaining figures in this paper histograms like these are summarised by five percentiles: the 5th, 25th, 50th (also known as the median), 75th, and 95th.

2 Decoding times

In each panel, the bottom graph shows for one code of each construction the median number of iterations to get a successful decoding as a function of E_b/N_0; upper and lower bars show the 5th, 25th, 75th and 95th percentiles of the number of iterations.

(a)

(b)

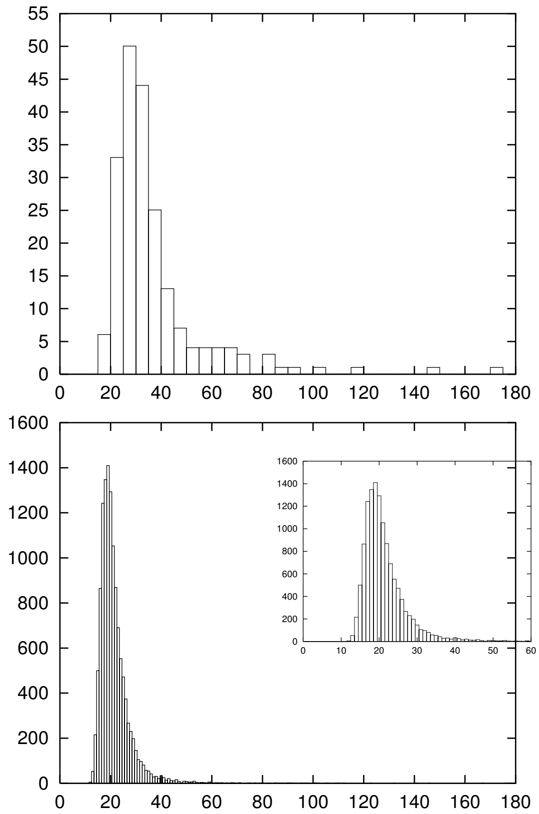


Figure 1: Histogram of number of iterations for a regular Gallager code with transmitted block length $N = 9972$ and rate $R = 1/2$. (a) Channel signal to noise ratio $x/\sigma = 1.15$. (b) $x/\sigma = 1.18$.

The large–iteration tail of histogram (b) appears to be quite well fitted by a power law $f(t) \propto 1/t^6$.

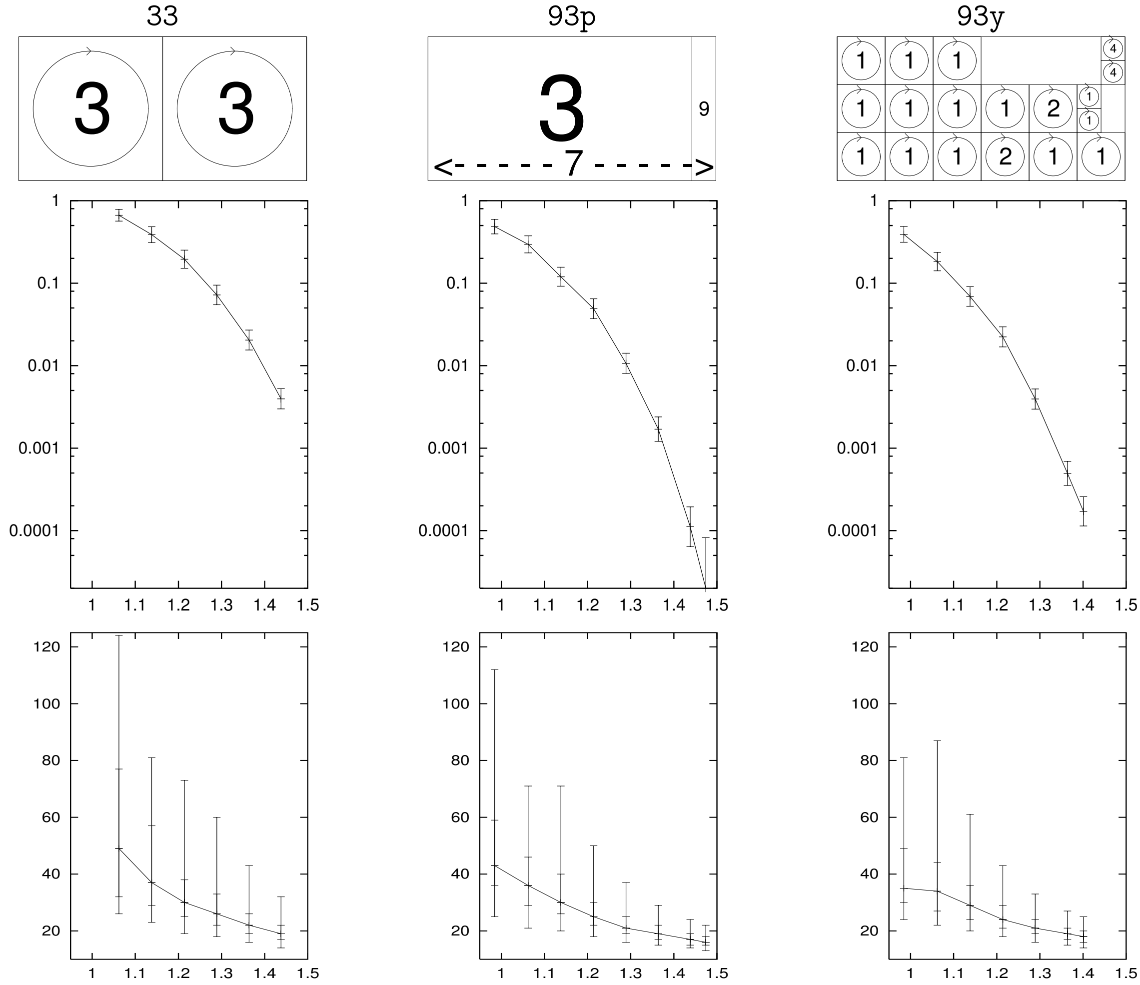


Figure 2: Upper panels: constructions of regular and irregular codes. Lower panels: performance of these codes. The construction types shown are regular, (`33`), Poisson (`93p`), and super–Poisson (`93y`).
Notation for upper panels for all constructions except `93p`: an integer represents a number of permutation matrices superposed on the surrounding square. Horizontal and vertical lines indicate the boundaries of the permutation blocks. Notation for the Poisson construction `93p`: integers '3' and '9' represent column weights. The integer '7' represents the row weight.
Lower panels show the performance of several random codes of each construction. Vertical axis: block error probability. Horizontal axis: E_b/N_0 in dB. All codes have $N = 9972$, and $K = M = 4986$.
All errors were detected errors, as is usual with Gallager codes.

(a)

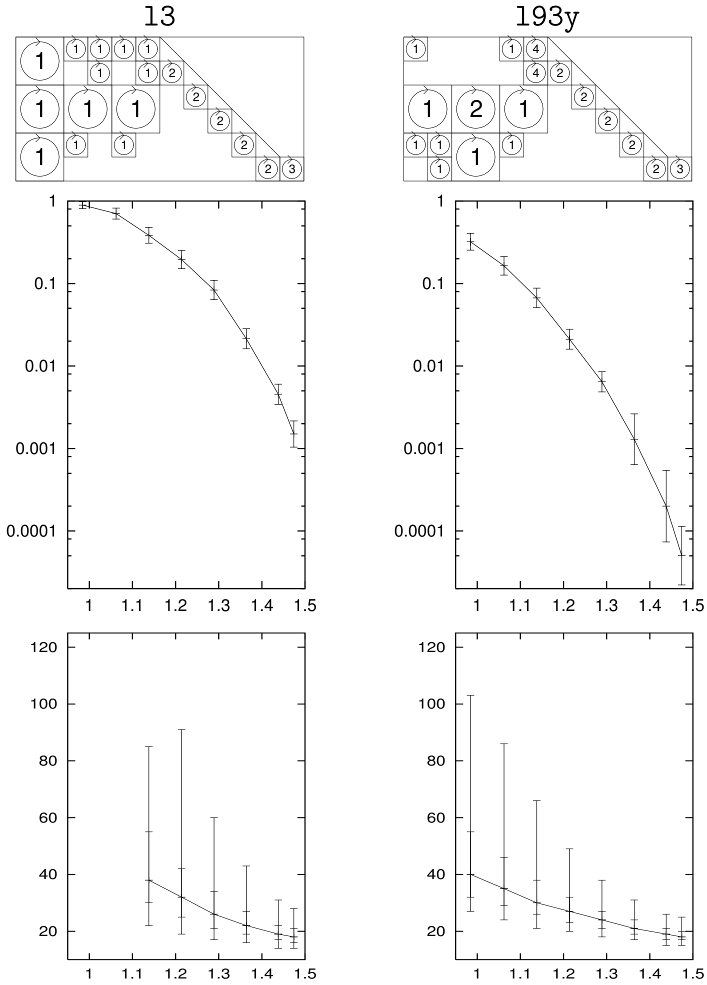
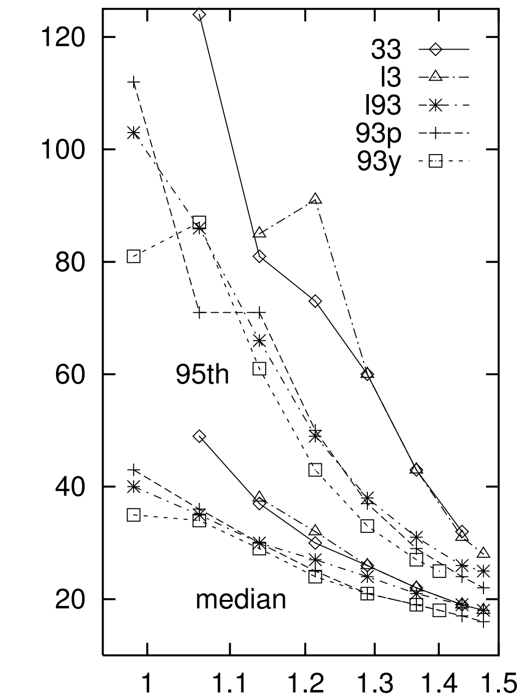
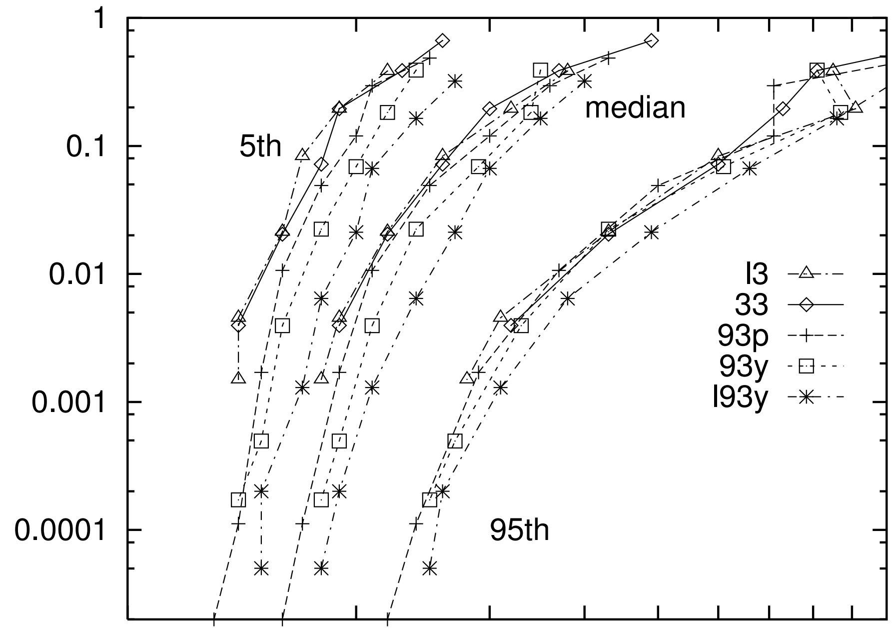


Figure 3: (a) Upper panels: construction methods l3 and l93y. As in figure 2, an integer represents a number of permutation matrices superposed on the surrounding square. A diagonal line represents an line of 1s. Horizontal and vertical lines indicate the boundaries of the permutation blocks. Lower pictures: Variability of performance among l3 and l93y codes. Vertical axis: block error probability. Horizontal axis: E_b/N_0 in dB.



(a)



(b)

Figure 4: (a) Number of iterations versus E_b/N_0. (b) Error probability versus number of iterations (on log scale). Curves show 5th, 50th and 95th percentiles for codes with five different constructions.

Figure 4 shows (a) the decoding times of all codes plotted against E_b/N_0, and (b) the error probability plotted against the decoding time. There are differences between codes because the codes have different absolute performances.

3 Discussion

The differences between constructions are much smaller than the range of decoding times within one construction.