Information Retrieval Using Hierarchical Dirichlet Processes

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Introduction

- This talk is about *ad-hoc* information retrieval.
- In other words, we are given...
  - A collection of documents, $\mathcal{C} = (\boldsymbol{d}_1, \boldsymbol{d}_2, \ldots)$.
  - A query, $\boldsymbol{q}$.
- Our task is to sort the collection in order of *relevance* to $\boldsymbol{q}$.
- The exact definition of relevance is open to interpretation.

Approaches To Information Retrieval

- There are a number of different approaches:
  - Vector space methods
  - ‘Traditional’ probabilistic models
  - Language modelling
    - Uses a statistical language model derived from the query and/or the document.
    - Relevance is defined based on the probability of the query / document under the model, or by comparing models.
- This work extends the language modelling framework.

‘Traditional’ And Vector Space Approaches

- A wide variety of different models, the most successful being BM25.
- Features common to many of the models in this category include:
  - tf.idf like weighting—terms appearing often in the document are more heavily weighted. Terms appearing in many documents are considered less important.
  - Document length normalisation—longer documents are more likely to contain query terms by chance.

Language Modelling Approaches

- Probabilistic models define a probability distribution over the set of all possible texts.

- The majority of methods use *bag of terms* models—The terms in the document are generated independently:

$$\Pr(\boldsymbol{x}) = \prod_{i=1}^{N} \Pr(x_i)$$

- Bayes' theorem can be used to invert the distributions.

Language Modelling Approaches

- There are three main approaches
  - One language model based on the query, used to construct documents.
  - One language model based on each document, used to construct the query.
  - Language models for both the query and the document, relevance defined by comparing the two (KL Divergence)

- Here we train using the documents rather than the queries—more data available.

Smoothing

- Training a language model on a single document/query gives poor performance. Models are *smoothed* by combining with a collection wide model:

$$P_s(x \mid d) = \gamma(x, d) P(x \mid d) + (1 - \gamma(x, d)) P_{\mathcal{C}}(x)$$

- Smoothing techniques include Jelineck-Mercer, absolute discounting, and (non-hierarchical) Dirichlet priors.

- $P_{\mathcal{C}}(x)$ is usually either the collection term frequency, or the document frequency. It must be specified *ab initio*.

The Dirichlet Distribution

- The collection model employed in this work will make use of the *hierarchical Dirichlet process*. But we'll begin by introducing a close relative, the *Dirichlet distribution*.
- The Dirichlet distribution is a probability distribution over probability distributions (conjugate to the multinomial).
- Samples are finite, discrete distributions, $\boldsymbol{p} = (p_1, p_2, \ldots)$.

The Dirichlet Distribution

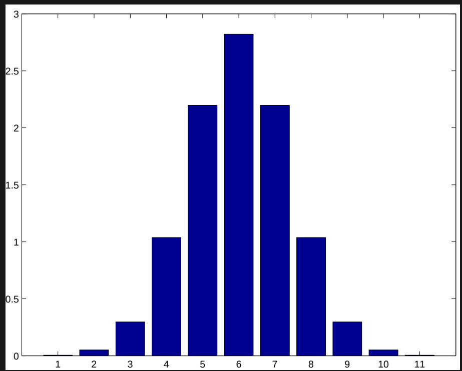
- The distribution is given by:

$$\Pr(\boldsymbol{p}) = \begin{cases} \frac{1}{Z(\gamma \boldsymbol{H})} \left(\prod_{i=1}^{N} p_i^{\gamma H_i - 1} \right) & \text{If } \sum_{i=1}^{N} p_i = 1 \\ 0 & \text{Otherwise} \end{cases}$$

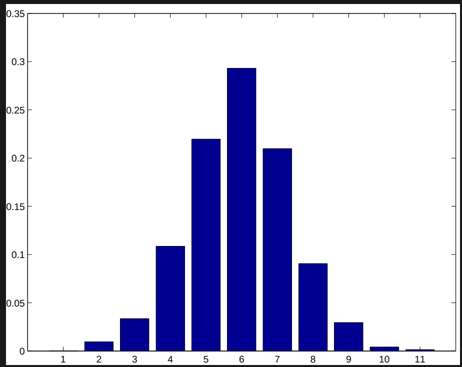
- $\boldsymbol{H} = (H_1, H_2, \ldots)$ is a *normalised base measure*, defining the mean of the distribution.

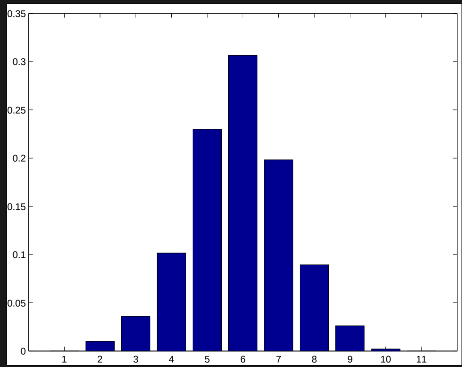
- γ is a *concentration parameter*—larger γ values give samples more tightly clustered around the mean.

Draws From A Dirichlet Distribution

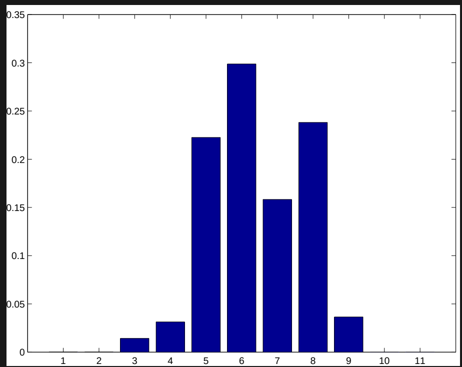


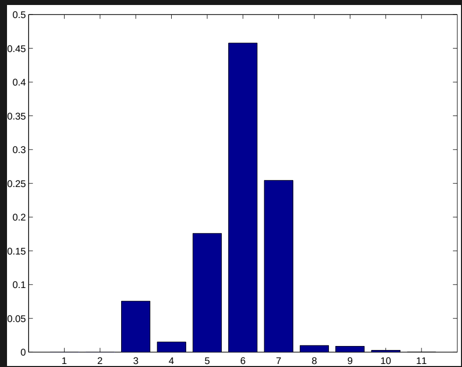
Base Measure





$\gamma = 1000$





$\gamma = 10$

Pòlya Urns

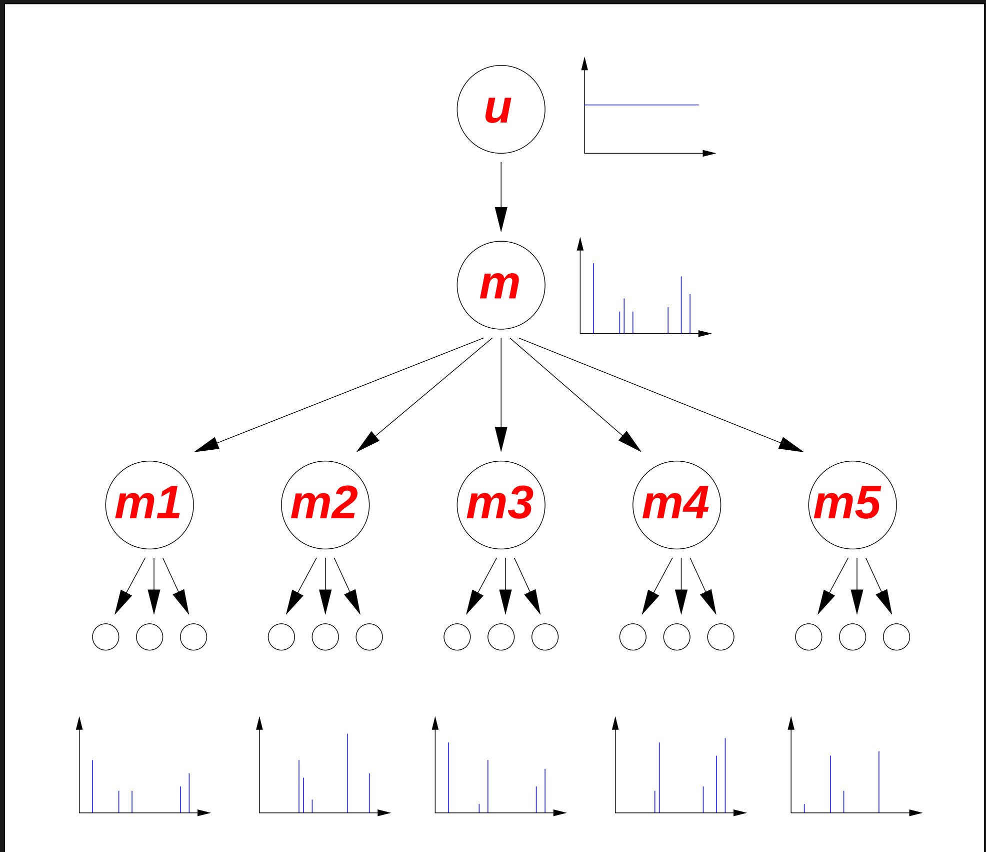
- We can sample explicitly from a Dirichlet distribution. Alternatively a sample can be obtained implicitly using a Pòlya urn scheme.

- Samples are obtained by drawing from an urn containing γH_1 balls of colour 1, γH_2 balls of colour 2 and so on...

- After each sample, the ball is returned, and *a new ball is added* of the same colour.

- The resulting set of samples are distributed according to a *single* sample from the Dirichlet distribution.

The Hierarchical Dirichlet Distribution



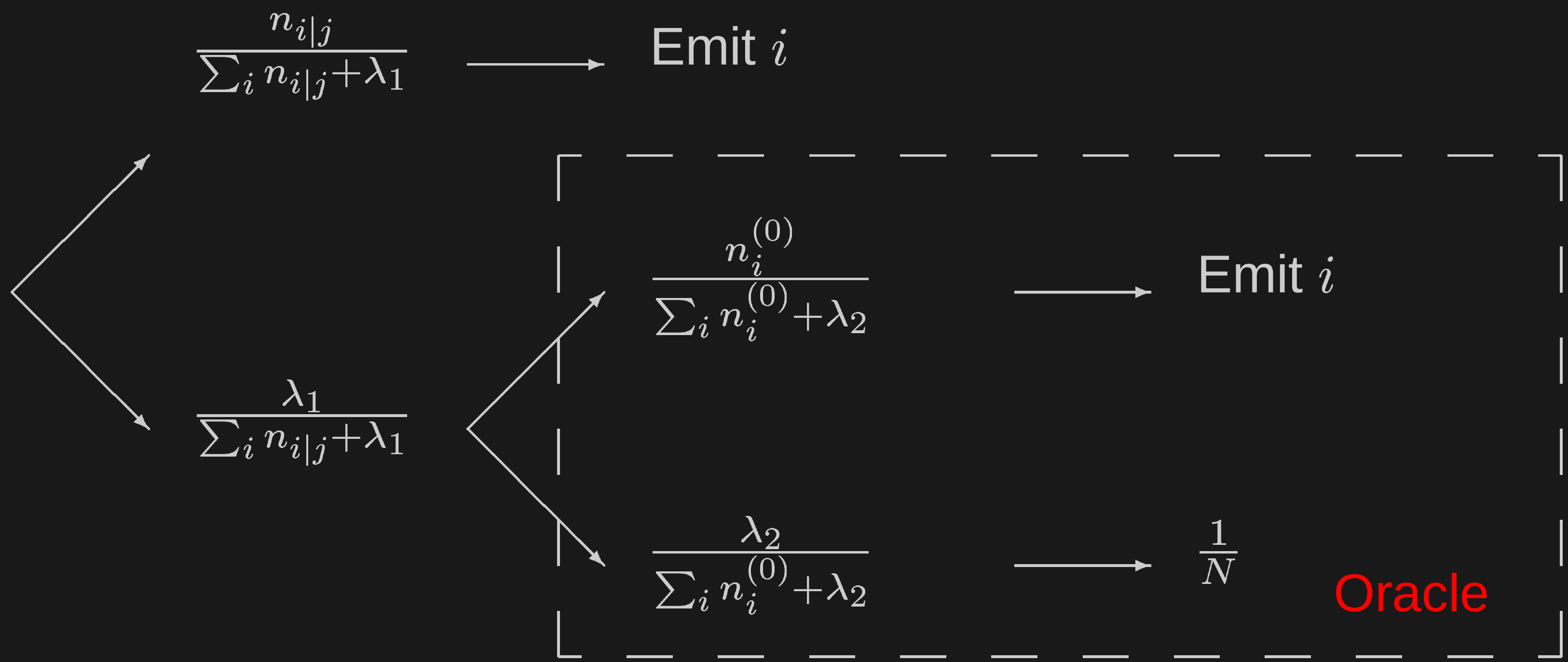
Each sample location has a label, y_i, giving the multinomial from which it is drawn:

$$\begin{aligned} x_i &\sim \text{Multinomial}\left(\boldsymbol{m}_{y_i}\right) \\ \boldsymbol{m}_y &\sim \text{Dirichlet}\left(\lambda_1 \boldsymbol{m}\right) \\ \boldsymbol{m} &\sim \text{Dirichlet}\left(\lambda_2 \boldsymbol{u}\right) \end{aligned}$$

Oracle Formulation

- The hierarchical version of the Pòlya Urn scheme is the *Oracle* framework (otherwise known as a *Chinese Restaurant Franchise*).
- With some probability, new samples are generated using a Pòlya urn local to the related multinomial.
- The remainder of the time, the oracle is asked, which has its own urn.
- The oracle is shared between all multinomials.

Oracle Formulation



The Infinite Limit

- The hierarchical Dirichlet process can be viewed as the infinite limit of the hierarchical Dirichlet distribution.
- Importantly, distributions are still discrete, but now over a countably infinite set of states. This allows (approximately) infinite vocabularies to be modelled.
- You can't sample directly from a hierarchical Dirichlet process, but indirect samples can still be obtained using the oracle formulation.
- (In fact, it makes very little difference whether we use the finite or infinite model, but the infinite model avoids the need to set the vocabulary size).

The Collection Model

- The hierarchical Dirichlet process allows us to specify a generative model of the collection.
- A ‘parent’ distribution over terms is first generated from a Dirichlet process with a uniform base measure and concentration parameter λ_2.
- A distribution is then created for each document in the collection, using the parent distribution as the base measure. and concentration parameter λ_1.
- Finally, documents are constructed by drawing terms from the corresponding distribution.

The Collection Model

- This is intuitively appealing, as it is reasonable to assume there is a common distribution (e.g. 'English'), about which the distributions for individual documents can vary to some extent.

- λ_1 governs the extent to which document distributions can vary from the base.

- By making the base distribution a random variable, rather than fixing it from the start, information can be exchanged between documents.

- (This is very similar technique to that used in many smoothed n-gram language models).

Information Retrieval

- To perform information retrieval, we assume that the query was generated from the same distribution as one of the documents.
- Relevance is defined as the probability that the distribution used belonged to the corresponding document:

$$R(\boldsymbol{d}, \boldsymbol{q}) = \log\left(\Pr\left(y_q = y_d \mid \boldsymbol{x}_q, \boldsymbol{y}_{\mathcal{C}}, \boldsymbol{x}_{\mathcal{C}}\right)\right)$$

- We can use the collection model to find this via Bayes' rule:

$$\Pr\left(y_q = y_d \mid \boldsymbol{x}_q, \boldsymbol{y}_{\mathcal{C}}, \boldsymbol{x}_{\mathcal{C}}\right) \propto \Pr\left(\boldsymbol{x}_q \mid y_q = y_d, \boldsymbol{y}_{\mathcal{C}}, \boldsymbol{x}_{\mathcal{C}}\right) \cdot \Pr\left(y_q = y_d \mid \boldsymbol{y}_{\mathcal{C}}, \boldsymbol{x}_{\mathcal{C}}\right)$$

Prior Distributions

- Note that we need to specify a prior over documents:

$$\Pr\left(y_q = y_d \mid \boldsymbol{y}_{\mathcal{C}}, \boldsymbol{x}_{\mathcal{C}}\right)$$

- In this work the prior is uniform—all documents are *a priori* equally likely to produce the query.

- However, it is possible to specify an arbitrary prior, for example to incorporate additional knowledge about the collection or the user.

An Important Approximation

- Using the oracle formulation is fine if you know how many times the oracle was asked when producing the data we have already seen.
- Unfortunately we don't know this—we need to marginalise over all possibilities, which is prohibitively expensive.
- To solve this problem, we assume that the oracle was asked the first time that each term is seen in each document, and never asked subsequently.
- (This is essentially the same approximation as 'update exclusion' in traditional language modelling).

A Few Minor Points

- We make the assumption that the query terms are independent given the collection and the query label.
- In other words, we ignore query terms which have been already seen. As the query is typically much shorter than the documents in the collection, this is fairly justified.
- The model was implemented using the LEMUR language modelling and information retrieval toolkit.

The Score Function

- Putting it all together...

$$\Pr\left(x_q^{(i)} \mid y_q\right) = \underbrace{\frac{\mathrm{tf}\left(x_q^{(i)}, y_q\right)}{N_{y_d} + \lambda_1}}_{\text{Internal}} + \underbrace{\frac{\lambda_1}{N_{y_q} + \lambda_1} \frac{\mathrm{df}\left(x_q^{(i)}\right)}{\sum_{x'} \mathrm{df}\left(x'\right) + \lambda_2}}_{\text{Oracle}}$$

$$= \frac{1}{N_{y_q} + \lambda_1}\left(\mathrm{tf}\left(x_q^{(i)}, y_q\right) + \lambda_1 \mathrm{mdf}\left(x_q^{(i)}\right)\right)$$

in which the *modified document frequency* is defined as

$$\mathrm{mdf}\left(x\right) \triangleq \frac{\mathrm{df}\left(x\right)}{\sum_{x'} \mathrm{df}\left(x'\right) + \lambda_2}$$

The Score Function

- Rearranging a bit, and taking logs

$$\log\left(\Pr\left(x_q^{(i)} \mid y_q\right)\right) = \log\left(\frac{1}{N_{y_q} + \lambda_1}\right) + \log\left(1 + \frac{\mathsf{tf}\left(x_q^{(i)}, y_q\right)}{\lambda_1 \mathsf{mdf}\left(x_q^{(i)}\right)}\right) + \text{const.}$$

- Ignoring the constant, and summing over all query terms,

$$R\left(\boldsymbol{d}, \boldsymbol{q}\right) = \sum_i \log\left(1 + \frac{\mathsf{tf}\left(x_q^{(i)}, y_d\right)}{\lambda_1 \mathsf{mdf}\left(x_q^{(i)}\right)}\right) + N_q \log\left(\frac{1}{N_{y_d} + \lambda_1}\right)$$

Interpretation Of Individual Terms

- The individual terms in the score function can easily be interpreted

$$\sum_i \log \left(1 + \frac{\text{tf}\left(x_q^{(i)}, y_d\right)}{\lambda_1 \text{mdf}\left(x_q^{(i)}\right)}\right)$$ Logarithmic tf.idf-like term weighting.

$$N_q \log \left(\frac{1}{N_{y_d} + \lambda_1}\right)$$ Overall document length normalisation

- Both of these are commonly found in other methods, and arise naturally from the hierarchical Dirichlet model.

- (Note that this can be regarded as a vector space model with an additional ‘global’ term).

Experimental Tests

- Performance was compared with other methods on TREC-7 and -8 *ad-hoc* tasks
- (50 queries, 528155 documents, binary relevance judgements)
- Other methods used were:
  - BM-25
  - Twenty-One (Per document language model)
  - KL Divergence (Document and query language models)
  - Hierarchical Dirichlet model

Experimental Tests

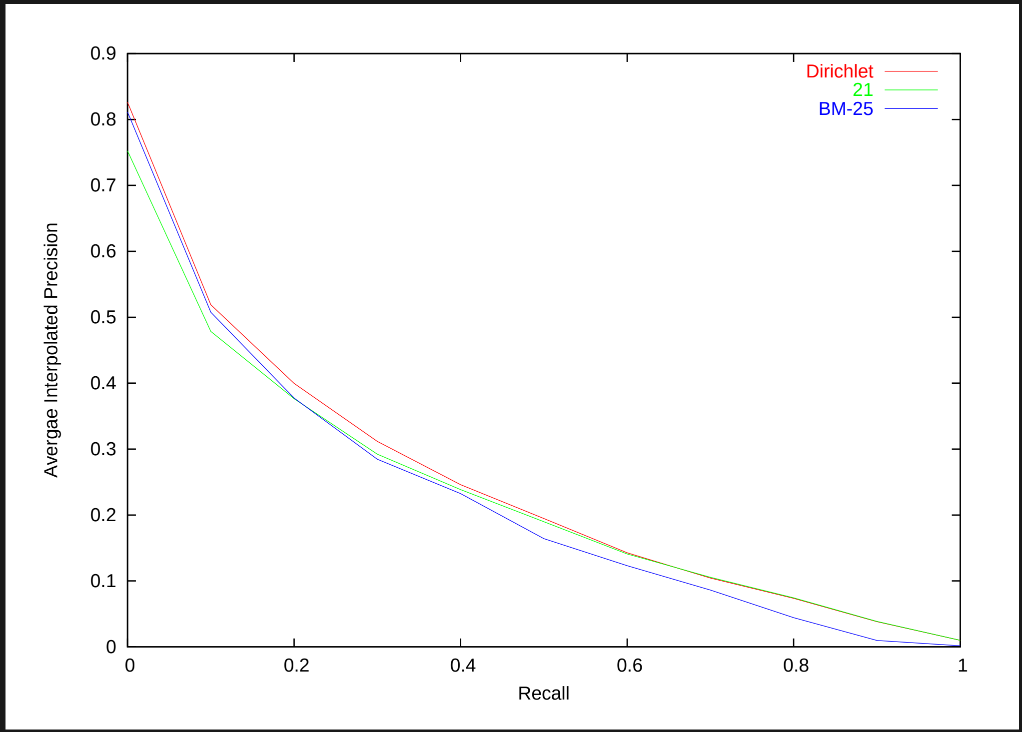
- Full query text (title, description and narrative) was used.
- The Dirichlet parameters were set to $\lambda_1 = 1250$ and $\lambda_2 = 750$.
- Preprocessing was limited to:
  - Basic stop word removal
  - Porter stemming

Results

Method	TREC-7	TREC-8
KL-Divergence	21.1%	25.7%
BM-25	21.5%	24.8%
Twenty-One	22.2%	26.2%
Dirichlet	23.3%	27.0%

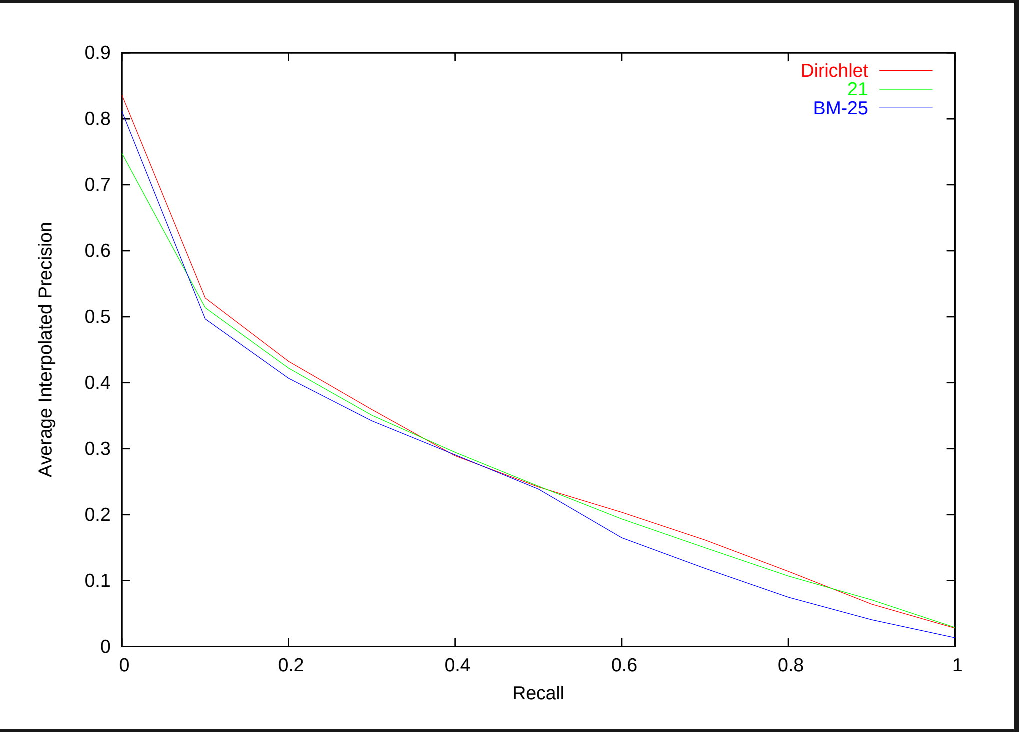
Average non-interpolated precision over top 1000 documents.

Results



Precision-Recall curves (TREC-7)

Results



Precision-Recall curves (TREC-8)

Further Work

- Relaxing the bag of terms assumption (n-gram models).
- Introducing collection structure (paragraph level, multiple collections etc.)
- Avoiding the oracle frequency approximation.
- Mixtures of hierarchies.
- Pitman-Yor processes.

Conclusions

- The hierarchical Dirichlet process can be successfully applied to whole collection modelling for information retrieval.
- By providing a generative model, the assumptions made by the model are made explicit.
- Whilst making minimal assumptions, the model can recover tf.idf like term weighting and document length normalisation.

That's All...

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http://www.inference.phy.cam.ac.uk/pjc51/