Delay the Teaching of Arithmetic?

Andrew M. Gleason
Professor of Mathematics Emeritus / Harvard University

I shall argue that the teaching of arithmetic algorithms (beyond one significant figure) should be postponed. For how long? I suggest until grade six, but this choice is only to give a rough indication of what I have in mind.

In the early '30s, L. P. Benezet, superintendent of schools in Manchester, New Hampshire, ran an experiment, delaying formal instruction in arithmetic in some public school classes and replacing it with practice in measuring, estimating, and above all, precise oral expression. The experiment ran for at least five years, and the results were excellent. A brief report appears in three installments in the *Journal of the National Education Association*, vol. 24 (1935), no. 8, 241−244, no. 9, 301−303, and vol. 25 (1936) no. 1, 7−8. The syllabus given in the second article indicates that there was no formal work in arithmetic until the sixth grade, yet, it is stated (first article), that by the end of one year, children who had started formal arithmetic in grade six had caught up with sixth graders who had started in grade three. In problem−solving, the experimental group was vastly superior to the control group, and when the children in the experiment reached high school, the English teachers were impressed by their facility in language. Even after allowing for the fact that the report is written entirely by the proponent, one cannot read the story without being convinced that a total restructuring of the mathematics program in elementary school should be given very serious thought.

Benezet's experience that sixth−graders with no prior instruction could catch up with students with three years of prior instruction deserves attention. If that is so, one can reasonably claim that the school time spent on arithmetic during grades 3, 4, and 5 is wasted.

At the Cambridge Conference in 1963, a distinguished applied mathematician, who had been brought up in another culture, said that he had been taught nothing about arithmetic, except how to count, until he was ten. The delay seems to have been no obstacle to his mathematical career. Since then I have often wondered why we are in such a hurry to start algorithmic instruction in the first grade. A child must learn to deal with money, it is said. If this is indeed the reason, why are they looking at numbers as large as those below? I was in high school before I had much occasion to deal with amounts of money requiring more than two digits, and three digits would have sufficed until college. (Because of inflation, today's children probably need one more digit than I did.)

When I was in the fifth grade we had a weekly "accuracy test" which always had just four problems in the following format:

```
  17083
  83540
  74097
  95301
  36648
  29441
+ 99328

 5478031
−3076422
```

```
87033
x 274


    _______
379 )159636
```

I cannot recall how long one had to do these problems.

There were five possible scores on these tests, 0, 25, 50, 75, 100, because there were no part scores, every problem was either right or wrong. I feel this was harsh. If a word is misspelled in an English composition, it does not make a whole paragraph wrong.

Consider what is involved in this test. For the addition problem, I counted 54 steps, for the subtraction, 11, for the multiplication, 43, and for the division, 35. (What constitutes a step is not clear; on another day I might count differently, to be sure; it matters little to the argument.) Suppose that a child understands the algorithm perfectly and has a 95% probability of getting any single step right: his probability of getting everything right would then be less than one−tenth of one percent and his expected mark on the test would be 23. Move the probability of a correct single step up to 99%, and the probability of a perfect paper becomes .24 and the expected mark 71. Bear in mind that the minimum passing mark was 75.

The model is admittedly simplistic; surely, steps are not all equally difficult, nor are they independent, and no account was taken of the possibility of a copying mistake (we had to copy the problems from the blackboard to our papers.) The basic questions remain, however. Is it reasonable to demand better than 99% accuracy of a ten−year−old in order to have a fair chance of getting even the minimum passing grade? Should a child wind up with a terrible mark of 50 for making two errors in 143 tries? I say "No!" to both questions.

It is plausible that children give up trying to understand mathematics problems because they know that efforts to understand a problem are likely to come to naught as a result of an arithmetic mistake. Some evidence in favor of this hypothesis is provided by an experiment carried out by SMSG in the '60s (I do not know the reference). Seventh−grade children deficient in mathematics were given aids (eg., hand−cranked calculators) that relieved them of the burden of long calculations so that they could concentrate on the structure of problems. They improved markedly during the year, not only in their ability to analyze problems, but also in their purely algorithmic skill!

Suppose we drop the arithmetic algorithms from the curriculum of grades 1 to 5. Children could still deal with problems germane to their own experience. They could learn to think primarily in terms of the plausibility of an answer. When a precise answer is clearly required they could use calculators and be taught that there is nothing wrong in doing so. By the age of 11 or 12 children would have sufficient experience with mathematical ideas to understand the algorithms and perhaps sufficient control of their own attention to get precise answers. According to Benezet, they do at least as well as students with several more years of instruction.

Such a major change in the curriculum would have to be accompanied by a major change in the style of teaching. I suspect that many grade−school teachers, uneasy themselves with mathematical ideas, take refuge in the simple right−wrong dichotomy and thereby avoid any serious effort to get students to think mathematically. A curriculum with little emphasis on precise calculation would force the teaching of ideas and the careful expression thereof. The required change in teaching style might easily prove to be the principal difficulty in making the change in the curriculum.

If we defer arithmetic instruction, what should we do instead? Benezet certainly had the right idea. Expose the children to situations involving mathematical ideas, insist that they learn to think clearly about these ideas, and avoid forcing them to carry out aversive calculations. I suggest that the set of mathematical ideas to be brought to children's attention should be enlarged considerably to conform to the needs of present−day society, but otherwise we should follow his blue−print closely. (I have some explicit ideas about the enlargement, and others may contribute theirs. Debate about the choice is not relevant here.)

Benezet Centre