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The Teaching of Arithmetic I: The Story of an experiment

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In the spring of 1929 the late Frank D. Boynton, superintendent of schools at Ithaca, New York, and president of the Department of Superintendence, sent to a number of his friends and brother superintendents an article on a modern public−school program. His thesis was that we are constantly being asked to add new subjects to the curriculum [safety instruction, health instruction, thrift instruction, and the like], but that no one ever suggests that we eliminate anything. His paper closed with a challenge which seemed to say, "I defy you to show me how we can cut out any of this material." One thinks, of course, of McAndrew's famous simile that the American elementary school curriculum is like the attic of the Jones' house. The Joneses moved into this house fifty years ago and have never thrown anything away.

I waited a month and then I wrote Boynton an eight−page letter, telling him what, in my opinion, could be eliminated from our present curriculum. I quote two paragraphs:

> In the first place, it seems to me that we waste much time in the elementary schools, wrestling with stuff that ought to be omitted or postponed until the children are in need of studying it. If I had my way, I would omit arithmetic from the first six grades. I would allow the children to practise making change with imitation money, if you wish, but outside of making change, where does an eleven−year−old child ever have to use arithmetic?
>
> I feel that it is all nonsense to take eight years to get children thru the ordinary arithmetic assignment of the elementary schools. What possible needs has a ten−year−old child for a knowledge of long division? The whole subject of arithmetic could be postponed until the seventh year of school, and it could be mastered in two years' study by any normal child.

Having written the letter, I decided that if this was my real belief, then I was falling down on the job if I failed to put it into practise. At this time I had been superintendent in Manchester for five years, and I had already been greatly criticized because I had dropped practically all of the arithmetic out of the curriculum for the first two grades and the lower half of the third. In 1924 the enrollment in the first grade was 20 percent greater than the enrollment in the second, because, roughly, one−fifth of the children could not meet the arithmetic requirements for promotion into the second grade and so were forced to repeat the year. By 1929 the enrollment of the first grade was no greater than that of the third.

Meanwhile, I was distressed at the inability of the average child in our grades to use the English language. If the children had original ideas, they were very helpless about translating them into English which could be understood. I went into a certain eighth−grade room one day and was accompanied by a stenographer who took down, verbatim, the answers given me by the children. I was trying to get the children to tell me, in their own words, that if you have two fractions with the same numerator, the one with the smaller denominator is the larger. I quote typical answers.

- "The smaller number in fractions is always the largest."
- "If the numerators are both the same, and the denominators one is smaller than the one, the one that is the smaller is the larger."
- "If you had one thing and cut it into pieces the smaller piece will be the bigger. I mean the one you could cut the least pieces in would be the bigger pieces."
- "The denominator that is smallest is the largest."
- "If both numerators are the same number, the smaller denominator is the largest − the larger − of the two."
- "If you have two fractions and one fraction has the smallest number at the bottom. It is cut into pieces and one has the more pieces. If the two fractions are equal, the bottom number was smaller than what the other one in the other fraction. The smallest one has the largest number of pieces − would have the smallest number of pieces, but they would be larger than what the ones that were cut into more pieces."

The average layman will think that this must have been a group of half−wits, but I can assure you that it is typical of the attempts of fourteen−year−old children from any part of the country to put their ideas into English. The trouble was not with the children or with the teacher; it was with the curriculum. If the course of study required that the children master long division before leaving the fourth grade and fractions before finishing the fifth, then the teacher had to spend hours and hours on this work to the neglect of giving children practise in speaking the English language. I had tried the same experiment in schools in Indiana and in Wisconsin with exactly the same result as in New Hampshire.

In the fall of 1929 I made up my mind to try the experiment of abandoning all formal instruction in arithmetic below the seventh grade and concentrating on teaching the children to read, to reason, and to recite − my new Three R's. And by reciting I did not mean giving back, verbatim, the words of the teacher or of the textbook. I meant speaking the English language. I picked out five rooms − three third grades, one combining the third and fourth grades, and one fifth grade. I asked the teachers if they would be willing to try the experiment. They were young teachers with perhaps an average of four years' experience. I picked them carefully, but more carefully than I picked the teachers, I selected the schools. Three of the four schoolhouses involved [two of the rooms were in the same building] were located in districts where not one parent in ten spoke English as his mother tongue. I sent home a notice to the parents and told them about the experiment that we were going to try, and asked any of them who objected to it to speak to me about it. I had no protests. Of course, I was fairly sure of this when I sent the notice out. Had I gone into other schools in the city where the parents were high school and college graduates, I would have had a storm of protest and the experiment would never have been tried. I had several talks with the teachers and they entered into the new scheme with enthusiasm.

The children in these rooms were encouraged to do a great deal of oral composition. They reported on books that they had read, on incidents which they had seen, on visits that they had made. They told the stories of movies that they had attended and they made up romances on the spur of the moment. It was refreshing to go into one of these rooms. A happy and joyous spirit pervaded them. The children were no longer under the restraint of learning multiplication tables or struggling with long division. They were thoroughly enjoying their hours in school.

At the end of eight months I took a stenographer and went into every fourth−grade room in the city. As we have semi−annual promotions, the children who had been in the advanced third grade at the time of the beginning of the experiment, were now in the first half of the fourth grade. The contrast was remarkable. In the traditional fourth grades when I asked children to tell me what they had been reading, they were hesitant, embarrassed, and diffident. In one fourth grade I could not find a single child who would admit that he had committed the sin of reading. I did not have a single volunteer, and when I tried to draft them, the children stood up, shook their heads, and sat down again. In the four experimental fourth grades the children fairly fought for a chance to tell me what they had been reading. The hour closed, in each case, with a dozen hands waving in the air and little faces crestfallen, because we had not gotten around to hear what they had to tell.

For some years I had noted that the effect of the early introduction of arithmetic had been to dull and almost chloroform the child's reasoning faculties. There was a certain problem which I tried out, not once but a hundred times, in grades six, seven, and eight. Here is the problem: "If I can walk a hundred yards in a minute [and I can], how many miles can I walk in an hour, keeping up the same rate of speed?"

In nineteen cases out of twenty the answer given me would be six thousand, and if I beamed approval and smiled, the class settled back, well satisfied. But if I should happen to say, "I see. That means that I could walk from here to San Francisco and back in an hour" there would invariably be a laugh and the children would look foolish.

I, therefore, told the teachers of these experimental rooms that I would expect them to give the children much practise in estimating heights, lengths, areas, distances, and the like. At the end of a year of this kind of work, I visited the experimental room which had had a combination of third− and fourth−grade children, who now were fourth and fifth graders. I drew on the board a rough map of the western end of Lake Ontario, the eastern end of Lake Erie, and the Niagara River. I asked them to guess what it was, and was not surprised when they identified the location. I then labeled three spots along the river with the letters "Q," "NF," and "B." They identified Niagara Falls and Buffalo without any difficulty, but were puzzled by the "Q." Some thought it was Quebec but others knew it was not. I finally told them that it was Queenstown. I then drew a cross section of the falls, showing the hard layer of rock above and the soft layer eating out underneath, and they told me what it was and why it was that the stone was falling, little by little, from the edge. They told me how this process was going on. I then made the statement that in 1680, when white men had first seen the falls, the falls were 2500 feet lower down than they are at present. I then asked them at what rate the falls were retreating upstream. These children, who had had no formal arithmetic for a year but who had been given practise in thinking, told me that it was 250 years since white men had first seen the falls and that, therefore, the falls were retreating upstream at the rate of ten feet a year. I then remarked that science had decided that the falls had originally started at Queenstown, and, indicating that Queenstown was now ten miles down the river, I asked them how many years the falls had been retreating. They told me that if it had taken the falls 250 years to retreat about a half mile, it would be at the rate of 500 years to the mile, or 5000 years for the retreat from Queenstown. The map had been drawn so as to show the distance from Niagara Falls to Buffalo as approximately twice the distance from Queenstown to Niagara Falls. Then I asked these children whether they had any idea how long it would be before the falls would retreat to Buffalo and drain the lake. They told me that it would not happen for another ten thousand years. I asked them how they got that and they told me that the map indicated that it was twenty miles from Niagara Falls to Buffalo, or thereabouts, and that this was twice the distance from Queenstown to Niagara Falls!

It so happened that a few days after this incident I was visiting a large New England city with five of my brother superintendents. Our host was interested in my description of this incident and suggested that I try the same problem on a fifth grade in one of his schools. With the other superintendents as audience, I stood before an advanced fifth grade in what was known as the Demonstration School, the school used for practise teaching and to which visitors were always sent.

> The home superintendent: Boys and girls, would you like to have Superintendent Benezet of Manchester, New Hampshire, ask you some questions about Niagara Falls?
>
> *The children express pleasure at the idea.*
>
> Mr. Benezet: [Drawing a map on the board] Children, what is this that I have drawn on the blackboard?

Children: The Great Lakes.
Mr. B.: Good. What lakes?
A child: Lake Ontario and Lake Erie.
Mr. B.: Good. What is the river?
Child: The St. Lawrence River.
Mr. B.: That is really correct. It is the St. Lawrence River. But they call it by a different name here. They call it the Niagara River. What have you heard in connection with the Niagara River?
Another child: Niagara Falls are there.
Another child: Niagara Falls are connected with Niagara River.
Mr. B.: Oh! How are they connected?
Child: The water trickles down the Falls and goes into the Niagara River.
Mr. B.: I should call that quite a trickle. Have any of you children seen Niagara Falls?

Three raise their hands.

Mr. B.: How high are the falls? Have you any idea? Are they higher than this room?
Children: Yes [dubiously].
Mr. B.: Well, how high is this room?

Its height is guessed anywhere from 11 feet to 40 feet. The room is actually about 16 feet high. The question of the height of the falls is finally dropped.

Mr. B.: Well, never mind how high the falls are. On this map here I have indicated one spot and marked it "NF," and another spot and marked it "B." What does "NF" mean?
Children: Niagara Falls.
Mr. B.: What does "B" stand for?
Another child: Bay.
Mr. B.: No. Remember that Niagara Falls is not only the name of the Falls, but the name of a city.
Child: Baltimore.

After considerable pause, the home superintendent, in the back of the room, tells the class that the name of the city is also the name of an animal.

Child: Buffalo.
Mr. B.: Yes. Now there is another town here that I am going to mark "Q." It is not Quebec; it is Queenstown. People who have studied this carefully tell us that once upon a time the falls were at Queenstown. Tell me now. What does it mean if I say that I show you the cross section of an apple?

Class is uncertain.

Mr. B.: Suppose that you cut an apple in half with a knife. What do I show you if I hold up one−half?
Child: Half the apple.
Another child: The core of the apple.
Third child: The inside of an apple.
Mr. B.: Tell me. Is the word "section" a new word to the majority of you?

Enthusiastic chorus of "No."

Mr. B.: Well, a cross−section of an apple means a cut right thru an apple. Why have I said this to you?

Meantime he has drawn on the board a cross−section of Niagara Falls.

Child: Because that is a cross−section of the falls.

Mr. Benezet now explains the two kinds of rock and asks which is the harder. They finally decide that the rock above is the harder. He then shows how the underneath rock rotted away, and that finally there was a shelf of hard rock overhanging. This became too heavy and fell off; and the falls have thereby moved back some ten feet.

Mr. B.: Now, when white men first saw the falls in 1680 [placing this date on the board], the falls were further down the river than they are now, and it is estimated that since that time they have moved back upstream about 2500 feet. Now how long ago was it that white men first saw the falls?
Child: Four hundred years.
Another child: Two hundred years.
Third child: Three hundred years.

Guesses range anywhere between 110 years and 450 years. One boy says it was about the time that Columbus sailed to America; another says that it was about the time of the Pilgrims and the Puritans.

Mr. B.: Well, how are we going to find out?

General bewilderment for a while. Finally:

Child: Take 1930 and subtract it from 1680.
Mr. B.: Fine.

He writes on the blackboard:

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1680
1930
−−−−
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Mr. B.: Now take a look and tell me how many years that was. See if you can tell me before we subtract it, figure by figure.

It is to be noted that not one child called attention to the wrong position of the two sets of figures. They guess 350 years, 200 years, 400 years.

Mr. B.: Well, let's subtract it figure by figure.
Child: Zero from 0 equals 0. Three from 8 equals 5. Nine from 6 equals 3. Three hundred fifty years is the answer.
Mr. B.: How many think that 350 years is right?

About two−thirds of the hands go up. Finally two or three think that it is wrong.

Mr. B.: All right, correct it.

> Child: It should have been 9 from 16 equals 7.
>
> *Mr. Benezet thereupon puts down 750 for the answer. When he asks how many in the room agree that this is right, practically every hand is raised. By this time the local superintendent was pacing the door at the rear of the room and throwing up his hands in dismay at this showing on the part of his prize pupils. After a time, as Mr. Benezet looks a little puzzled, the children gradually become a little puzzled also. One little girl, Elsie Miller, finally comes to the board, reverses the figures, subtracts, and says the answer is 250 years.*
>
> Mr. B.: All right. If the falls have retreated 2500 feet in 250 years, how many feet a year have the falls moved upstream?
> Child: Two feet.
>
> *Mr. Benezet registers complete satisfaction and asks how many in the class agree. Practically the whole class put hands up again.*
>
> Mr. B.: Well, has anyone a different answer?
> Child: Eight feet.
> Another child: Twenty feet.
>
> *Finally Elsie Miller again gets up, and says the answer is ten feet.*
>
> Mr. B.: What? Ten feet? *(Registering great surprise)*
>
> *The class, at this, bursts into a roar of laughter. Elsie Miller sticks to her answer, and is invited by Mr. Benezet to come up and prove it. He says that it seems queer that Elsie is so obstinate when everyone is against her. She finally proves her point, and Mr. Benezet admits to the class that all the rest were wrong.*
>
> Mr. B.: Now, what fraction of a mile is it that the falls have retreated during the last 250 years?
>
> *Children guess 3/2, 3/4, 2/3, 1/20, 7/8 − everything except 1/2. The bell for dismissal rings and the session is over.*

It will be noted that the local superintendent gave them a little hint at the outset, that was not given to the Manchester children, when he said, "Niagara Falls." They were prepared to identify my map. Also, the Manchester children who had not learned tables but had talked a great deal about distances and dimensions, recognized the fact that 2500 feet was about a half a mile, while the children in the larger city who were fresh from their tables, had little conception of the distance.

I was so delighted with the success of the experiment so far that in the fall of 1930 we started six or seven other rooms along the same line. The formal arithmetic was dropped and emphasis was placed on English expression, on reasoning, and estimating of distances.

One day I tried an experiment having to do with English expression. I hung before a 7−B class a copy of a painting by Frederick Waugh, representing a polar bear floating on a small berg of ice. This was a traditionally taught room in a school where there were very few children of foreign extraction. I asked the children to write anything which they felt inspired to put down as a result of seeing the picture. Three quarters of an hour later I hung the same picture before another 7−B grade, one of the experimental groups this time, in a school where not more than three children in the room came from homes where English was the language of the parents. I then called the seventh−grade teachers of the city together and read them the ten best papers from one room and the ten best from the other. I asked them if they saw any difference. One teacher remarked that one group was about a year and a half or two years ahead of the other in maturity of expression, and there was general assent to this statement. I said to the teachers, "If I should tell you that one group came from the "A" school and the other from the "B," from which school would you guess the better group of papers came?"

"Oh, the "A" school, undoubtedly," said they, naming the school whose patrons speak English in their homes.

"Well," I said, "it was just the other way," and there was a murmur of incredulity. Then we analyzed the papers and counted the number of adjectives used by the traditionally taught pupils. There were forty all told: nice, pretty, blue, green, cold, etc. We then counted the adjectives used by the other group [the number of papers was approximately the same] and we found 128, including magnificent, awe−inspiring, unique, majestic, etc. The little Greeks, Armenians, Poles, and French−Canadians had far surpassed their English−speaking opponents.

I next tried a rather similar test. I hung the same picture − a landscape representing a river scene in the vicinity of Manchester − before ten different fifth−grade rooms. Five of them had been brought up under the old traditional curriculum and five of them were of the experimental group. It was the same story: the experimental rooms far excelled the others in fluency of expression. They used words that the others had never heard of. Nevertheless, when we came to test the papers for spelling, the poorest of the experimental rooms exactly tied the record of the best of the traditional groups. The most surprising result came in a certain room in which there was housed a 5−B grade and a 5−A. The younger pupils, the 5−B's, had been brought up under the experimental curriculum, without arithmetic, while the other half of the room were traditional. The 5−A's made the poorest record of all the ten groups while the 5−B's, the younger group, were next to the top. For four months they had been taught by the same teacher but by different methods.

Now we were ready to experiment on a much larger scale. By the fall of 1932 about one−half of the third−, fourth−, and fifth−grade rooms in the city were working under the new curriculum. Some of the principals were a little dubious and asked permission to postpone formal arithmetic until the beginning of the sixth grade instead of the beginning of the seventh. Accordingly, permission was given to four schools to begin the use of the arithmetic book with the 6−B grade. About this time Professor Guy Wilson of Boston University asked permission to test our program. One of our high school teachers was working for her master's degree at Boston University and as part of her work he assigned her the task of giving tests in arithmetic to 200 sixth grade children in the Manchester schools. They were divided fairly evenly, 98 from experimental rooms and 102 from the traditional groups, or something like that. These were all sixth graders. Half of them had had no arithmetic until beginning the sixth grade and the other half had had it throughout the course, beginning with the 3−A. In the earlier tests the traditionally trained people excelled, as was to be expected, for the tests involved not reasoning but simply the manipulation of the four fundamental processes. By the middle of April, however, all the classes were practically on a par and when the last test was given in June, it was one of the experimental groups that led the city. In other words these children, by avoiding the early drill on combinations, tables, and that sort of thing, had been able, in one year, to attain the level of accomplishment which the traditionally taught children had reached after three and one−half years of arithmetical drill. [This article will be continued in the December issue.]

Part II | Part III
Benezet Centre

The Teaching of Arithmetic II: The Story of an Experiment

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[This is the second instalment of an article describing an experiment which has been carried out in Manchester, New Hampshire, since 1929. In the preceding section, which appeared in the November Journal, Mr. Benezet explained that: In some schools of Manchester, the only arithmetic in the first six grades was practise in estimating heights, areas, and the like; formal arithmetic was not introduced until the seventh grade. In tests given to both the traditionally and experimentally taught groups, it was found that the latter had been able in one year to attain the level of accomplishment which the traditionally taught children had reached after three and one−half years of arithmetic drill. In addition, because the teachers in the experimental group had had time to concentrate on teaching the children to "read, reason, and recite," these children developed more interest in reading, a better vocabulary, and greater fluency in expression.]

In the fall of 1933 I felt that I was now ready to make the big plunge. I knew that I could defend my position by evidence that would satisfy any reasonable person. Accordingly, a committee of our principals drew up a new course of study in arithmetic. I would have liked to go the whole route and drop out all the arithmetic until we reached the seventh grade, for we had proved, in the case of four rooms, that this could be done without loss, but the principals were more cautious than I was and I realized, too, that I would now have to deal with the deeply rooted prejudices of the educated portion of our citizens. Therefore, a compromise was reached. Accordingly, on September 1, 1933, we handed out the following course of study in arithmetic:

- ***Grade I*** − There is no formal instruction in arithmetic. In connection with the use of readers, and as the need for it arises, the children are taught to recognize and read numbers up to 100. This instruction is not concentrated into any particular period or time but comes in incidentally in connection with assignments of the reading lesson or with reference to certain pages of the text.

  Meanwhile, the children are given a basic idea of comparison and estimate thru [sic] the understanding of such contrasting words as: more, less; many. few; higher, lower; taller, shorter; earlier, later; narrower, wider; smaller, larger; etc.

  As soon as it is practicable the children are taught to keep count of the date upon the calendar. Holidays and birthdays, both of members of the class and their friends and relatives, are noted.

- ***Grade II*** − There is no formal instruction in arithmetic.

  The use of comparatives as taught in the first grade is continued.

  The beginning is made in the telling of time. Children are taught to recognize the hours and half hours.

The recognition of page numbers is continued. The children are taught to recognize any numbers that they naturally encounter in the books used in the second grade. If any book used in this grade contains an index, the children are taught what it means and how to find the pages referred to. Children will naturally pick up counting in the course of games which they play. They will also easily and without formal instruction learn the meaning of "half," "double," "twice," or "three times." The teacher will not devote any formal instruction to the meaning of these terms if the children do not pick them up naturally and incidentally.

To the knowledge of the day of the month already acquired is added that of the name of the days of the week and of the months of the year.

The teacher learns whether the children come in contact with the use of money at all in their life outside the school. If so, the meaning of "penny," "nickel," "dime," and "dollar" is taught. In similar fashion, and just incidentally, the meaning and relation of "pint" and "quart" may be taught.

- ***Grade III*** − While there is no formal instruction in arithmetic, as the children come across numbers in the course of their reading, the teacher explains the significance of their value.

  Before the year is over the children will be taught that a "dime" is worth 10 cents, and a "dollar" 10 dimes or 100 cents, a "half dollar" 5 dimes or 50 cents, etc. They will learn that 4 quarters, or 2 halves, are worth as much as one dollar.

  They add to their knowledge of hours and half hours the ability to tell time at any particular moment. The first instruction omits such forms as 10 minutes to 4; or 25 minutes to 3. They are first taught to say 3:50; 2:35; etc. In this connection they are taught that 60 minutes make one hour.

  It is now time, also, for them to know that 7 days make a week and that it takes 24 hours to make a day. They are also taught that there are 12 months in a year and about 30 days in a month.

  The instruction in learning to count keeps pace with the increasing size of the textbooks used and the pages to which it is necessary to refer. Games bring in the recognition of numbers. Automobile license numbers are a help in this respect. For example, the teacher gives orally the number of a car [of not over four digits] which most of the children are likely to see, and later asks for the identification of the car. Children are encouraged to bring to class their own house numbers, automobile license numbers, or telephone numbers and invite the class to identify them.

  The use of comparisons is continued, especially those involving such relations as "half," "double," "three times," and the like.

- ***Grade IV*** − Still there is no formal instruction in arithmetic.

  By means of foot rules and yard sticks, the children are taught the meaning of inch, foot, and yard. They are given much practise in estimating the lengths of various objects in inches, feet, or yards. Each member of the class, for example, is asked to set down on paper his estimate of the height of a certain child, or the width of a window, or the length of the room, and then these estimates are checked by actual measurement.

  The children are taught to read the thermometer and are given the significance of 32 degrees, 98.6 degrees, and 212 degrees.

  They are introduced to the terms "square inch," "square foot," and "square yard" as units of surface measure.

With toy money [or real coins, if available] they are given some practise in making change, in denominations of 5's only. All of this work is done mentally. Any problem in making change which cannot be solved without putting figures on paper or on the blackboard is too difficult and is deferred until the children are older.

Toward the end of the year the children will have done a great deal of work in estimating areas, distances, etc., and in checking their estimates by subsequent measuring. The terms "half mile," "quarter mile," and "mile" are taught and the children are given an idea of how far these different distances are by actual comparisons or distances measured by automobile speedom−eter.

The table of time, involving seconds, minutes, and days, is taught before the end of the year. Relation of pounds and ounces is also taught.

- ***Grade V−B*** − There still is no formal instruction in arithmetic except that the children are asked to count by 5's, 10's, 2's, 4's, and 3's. This work is done mentally at first with no written figures before them, either on paper or on the blackboard. This leads naturally to the multiplication tables of 5's, 10's, 2's, 4's, and 3's which, in this order, are given to the children before the end of the semester.

  With toy money, or with real coins if available, the children practise making change in amounts up to a dollar, involving, this time, the use of pennies.

  The informal work of previous grades in the estimating of distance, area, time, weights, measure of capacity, and the like, is continued. The ability to guess and estimate by games is developed. Each child in the class writes his estimate before these are checked up by actual measurement.

  The children compare the value of fractions and discover for themselves that 1/3 is smaller than 1/2 and greater than 1/4; i.e., that the larger the denominator the smaller the fraction. This is illustrated concretely or by pictures.

  Toward the end of the semester the children are given the book, Practical Problems in Mental Arithmetic, grade IV. The solution of these problems involves a knowledge of denominations which the children have not had and the use of tables and combinations which have not yet been taught to them. Nevertheless, children with a natural sense of numbers will be able to give the correct answers. The teacher will not take time to explain by formula or tables the solution of any problem to those who do not grasp it quickly and naturally. The purpose of the mental arithmetic book is to stimulate quick thinking and to get children away from the old−time method of using the fingers to do the work of the head. If some of the children do not grasp the problems easily and quickly, the teacher simply passes on, knowing that the power to reason will probably develop in them a year or two subsequently. The one thing which is avoided is that children shall get the idea that a fixed method or formula can be used as a substitute for thinking. The problems listed under September, October, and November are covered before the end of the semester.

- ***Grade V−A*** − The children are asked to count by 6's, 7's, 8's, and 9's. This work is done mentally without written tables before them, either upon paper or on the blackboard. After a time this leads naturally to the multiplication tables of 6's, 7's, 8's, and 9's. The attention of the children is called to the fact that in the table of 9's the second digit is always diminished by one [18, 27, 36, etc.] and the reason is explained that adding 9 is the same as adding 10 and taking away 1. In similar fashion it is shown that adding 8 is the same as adding 10 and taking away 2, so that in the table of 8's the second digit of each successive product is 2 less than the second digit of the product above it [48, 56, 64]. In similar fashion it is shown that adding 7 is the same as adding 10 and taking away 3. After the tables have been learned the teacher makes sure that the children know the products in any order; i.e., that it is not necessary for the child to start at the beginning of the table and run through until he reaches the product which he is asked to give. They learn that 2 times 3 is always equal to 3 times 2.

Children are given a little idea about the relative value of the fractions 1/2, 1/4, 1/5, and 1/10. Concrete examples assist in this; e.g., when the children remember that 2 quarters are worth one half dollar, it is easy to show them that twice 1/4 equals 1/2, or that twice 1/10 equals 1/5.

The problems listed under December to June, inclusive, in the book Practical Problems in Mental Arithmetic, grade IV, are covered in the course of the semester. If the children do not grasp the problem quickly and easily, the teacher does not stop to explain the method or prescribe any formula for solution. Of course as new terms occur in the problems [pecks, gallons, etc.] the teacher explains, incidentally, what they mean.

- ***Grade VI−B****[20 to 25 minutes a day]* − At this grade formal work in arithmetic begins. The first 108 pages of the Strayer−Upton Arithmetic, book III, are used as a basis.

  The processes of addition, subtraction, multiplication, and division are taught. Care is taken to avoid purely mechanical drill. Children are made to understand the reason for the processes which they use. This is especially true in the case of subtraction. Problems involving long numbers which would confuse them are avoided. Accuracy is insisted upon from the outset at the expense of speed or the covering of ground, and where possible the processes are mental rather than written. Before starting on a problem in any one of these four fundamental processes, the children are asked to estimate or guess about what the answer will be and they check their final result by this preliminary figure. The teacher is careful not to let the teaching of arithmetic degenerate into mechanical manipulation without thought.

  Fractions and mixed numbers are taught in this grade. Again care is taken not to confuse the thought of the children by giving them problems which are too involved and complicated.

- ***Grade VI−A****[25 minutes a day]* − The work of this grade is based upon Chapter II [pages 109 to 182] of the Strayer−Upton Arithmetic, book III, and the first 50 pages of book IV.

  Multiplication tables and tables of denominate numbers, hitherto learned, are reviewed. The teacher keeps in mind that the objectives to be gained are first of all reasoning and estimating, rather than mere ease in manipulation of numbers.

  Again, as in the previous grade, the children before beginning any problem make an estimate [individually] as to what the answer ought to be and check the final result by the preliminary guess.

- ***Grade VII−B****[25 minutes a day]* − The assignment in the text is the latter part of Strayer−Upton, book IV, beginning with page 51.

  Tables of denominate numbers, including United States money, found in the rear of book IV are reviewed. In addition to the table of linear measure, as given, it is taught that there are 1760 yards in a mile, 880 yards in a half mile, 440 yards in a quarter mile, etc.

  The teacher will omit any problems in the book which, because of the length of numbers involved, cause the child in using the four fundamental processes to lose sight of the reasoning process which, after all, is the main purpose of the problem.

There is a great deal of work in mental arithmetic, involving the solution of problems without reference to paper or blackboard. This is far more important than accuracy in the four fundamental processes.

- ***Grade VII−A****[30 minutes a day]* − The assignment in the text is the first one hundred pages of Strayer−Upton, book V, omitting the following pages: 1−10, 28, 71−77. Wherever possible the work is done mentally.

  Note that most of the pages omitted in this grade reoccur in book VI.

  The practise of estimating the probable answer and checking the result with this preconceived estimate is constantly followed.

  Again the teachers remember that ability to reason the problem correctly is far more important than errorless manipulation of the four fundamental processes.
- ***Grade VIII−B****[30 minutes a day]* − The assignment in the test is the latter part of Strayer−Upton, book V, beginning with page 101 [omitting pages 127−34] and the first 32 pages of book VI.

  The practise of making a preliminary estimate or an approximation at the answer before attacking the problem is continued. The ability to guess closely and promptly what the answer will be is one of the most important objectives to be gained from the study of arithmetic.

  Tables of denominate numbers are kept fresh in the minds of the children. The practise of estimating lengths, heights, and areas of familiar objects and the checking up by actual measurement is constantly kept up.
- ***Grade VIII−A****[30 minutes a day]* − The text for the grade is book VI of the Strayer−Upton series, beginning with page 35 and omitting the following pages: 36, 46−8, 57−9, 80−2, 92−3, 104, 158−188, 194, 203−4, 206−8.

  The work of this grade must necessarily be a summary of everything that has been learned in arithmetic, but, above all, the ability to approximate and estimate in advance the probable answer is kept as the important objective.

  The children are shown reasons for the various processes employed; why it is that a correct answer is obtained in the division of fractions by inverting the divisor and multiplying, etc. The ability to read problems intelligently and explain how they should be attacked is far more important than the ability to add large columns of figures without an error.

  The teacher will bear in mind that a great deal of work in mensuration [pages 88 to 100 inclusive] will be difficult for some pupils to understand. Of course this work is really using geometrical formulas without giving the geometrical reasons why they work, and some children will be unable to grasp the meaning of it all. It will be found worthwhile to have models in class and to perform experiments like filling a cylinder with water from 3 times the contents of a cone of equal base and altitude, etc.

  Again as much of the work as possible is done mentally. Problems are chosen to illustrate principles and give practise in reasoning rather than practise in the manipulation of large figures or complicated fractions.

Part I | Part III
Benezet Centre

The Teaching of Arithmetic III: The Story of an Experiment

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This is the third and final instalment of an article by Superintendent L. P. Benezet, in which he describes an experiment in arithmetic in the Manchester, New Hampshire, schools. The first installments [November 1935, p. 241−4 and December 1935, p. 301−3] have aroused many favorable comments. William McAndrew calls the material "powerful good reading, a scientific article free of the common dullness of such." Helen Ives Schermerhorn, of New Jersey, writes that upon returning to teach in junior high school after many years in the adult education field, she "was appalled at the changes which had taken place, the great number of new activities which had developed, each good in itself, but nevertheless cluttering up the time of the children. The weakness in English seemed inexcusable; too little time had been given to its mastery. I hope great things from the influences of Mr. Benezet's article. A letter from C. E. Birch, superintendent of schools, Lawrence, Kansas, indicates that the Lawrence schools have been revising the arithmetic program for the past two years. Mr. Birch has recommended the discussion in faculty meetings of the Benezet articles and their possible application in the light of the local situation.

Is your school making similar use of these articles? It would be an interesting thing to call some of the leading citizens in your community together around the table and read the articles to them to see what their attitude would be.

It must be understood that I knew very well what my hardest task was ahead. I had to show my more conservative teachers what we were trying to do and convert them to the idea that it could be done. I went into room after room, day after day, testing, questioning, giving out examples.

We had visitors. Two Massachusetts superintendents, a superintendent of a large Massachusetts city with five of his principals, and two instructors in the Boston Normal School came. They saw what we were trying to do and were surprised at the ability to reason and to talk, shown by children whose minds had not been chloroformed by the dull, drab memorizing of tables and combinations. But there were murmurs throughout the city. It finally broke out in a board meeting. A motion was made that we throw out the new course of study in arithmetic and go back to the old. It was defeated by a vote of nine to four, but a committee of three was appointed to study the problem carefully. Taking with me two members of the committee and a stenographer, I visited four different schools in our own city and three in a city not thirty miles away.

The most convincing test was in connection with the problem which I tried out in not less than six different rooms. Four of these rooms were made up of children who learned their arithmetic in the old formal way, whereas the other two were groups who had been taught according to the new method. In every case it was an advanced fifth grade, within one month of promotion to the 6−B.

I give verbatim accounts of two of these recitations, the first from a traditional room and the other from one of the experimental groups. I drew on the board a little diagram and spoke as follows: "Here is a wooden pole that is stuck in the mud at the bottom of a pond. There is some water above the mud and part of the pole sticks up into the air. One−half of the pole is in the mud; 2/3 of the rest is in the water; and one foot is sticking out into the air. Now, how long is the pole?"

> First child: "You multiply 1/2 by 2/3 and then you add one foot to that."
>
> Second child: "Add one foot and 2/3 and 1/2."
>
> Third child: "Add the 2/3 and 1/2 first and then add the one foot."
>
> Fourth: "Add all of them and see how long the pole is."
>
> Next child: "One foot equals 1/3. Two thirds divided into 6 equals 3 times 2 equals 6. Six and 4 equals 10. Ten and 3 equals 13 feet."

You will note that not one child saw the essential point, that 1/2 the pole was buried in the mud and the other half of it was above the mud and that 1/3 of this half equaled one foot. Their only thought was to manipulate the numbers, hoping that somehow they would get the right answer. I next asked, "Is there anybody who knows some way to get the length?"

> Next child: "One foot equals 3/3. Two−thirds and 1/2 multiplied by 6."

My next question was, "Why do you multiply by 6?"

The child, making a stab in the dark, said, "Divide."

It may be that he detected in my voice some stress on the word "multiply." I then gave them a hint which, had they been able to reason at all, should have shown them how to solve the problem. "How much of the pole is above the mud?" said I. The answer which I had hoped for was, of course, "One−half of it is above the mud."

> The first child answered: "One foot and 2/3."
>
> I looked dubious, so the second child said, "One foot and 1/3."

I then said, "I will change my question. How much of the pole is in the mud?"

> "Two−thirds," said the first child.
>
> "One−half," said the second.
>
> "One−half," said the third.

"Then how much of the pole is above the mud," said I thinking that now the answer was plainly indicated as one−half.

> "Two−thirds," said the next child.
>
> "One foot and 2/3," said the next.

"One−half of the pole is in the mud," said I. "Now, how long is the pole?" and the answers given were "Two feet." One and one−half feet." "One−half foot." "One foot." "One foot." "One foot," and I gave it up.

I gave the same problem the same week to a fifth grade in our city which had been brought up under our new curriculum, with no formal drill in addition, multiplication, and division of big numbers but with much mental work in reasoning. I drew the diagram again and said, "Here is a pond with a rock bottom and mud and water, with a pole sticking in the mud. One−half of the pole is in the mud; 2/3 of the rest of the pole is in the water; one foot of the pole sticks up in the air above the water. How long is the pole? How would you go to work to do that problem?"

First child: "You would have to find out how many feet there are in the mud."

"And what else?" said I.

Another child: "How many feet in the water and add them together."

"How would you go to work and get that?" said I to another child.

> "There are 3 feet in a yard. One yard is in the mud. One yard equals 36 inches. If 2/3 of the rest is in the water and one foot in the air [one foot equals twelve inches] the part in the water is twice the part in the air so that it must be 2 feet or 24 inches. If there are 3 feet above the mud and 3 feet in the mud it means that the pole is 6 feet or 72 inches long. Seventy−two inches equals 2 yards."

It amazed me to see how this child translated all the measurements into inches. As a matter of fact, to her, the problem was so simple and was solved so easily, that she could not believe that she was doing all that was necessary in telling me that the pole was 6 feet long. She had to get it into 72 inches and 2 yards to make it hard enough to justify my asking such a problem.

The next child went on to say, "One−half of the pole is in the mud and 1/2 must be above the mud. If 2/3 is in the water, then 2/3 and one foot equals 3 feet, plus the 3 feet in the mud equals 6 feet."

The problem seemed very simple to these children who had been taught to use their heads instead of their pencils.

The committee reported to the board and the board accepted their report, saying that the superintendent was on the right track. They merely suggested that, to quiet the outcry of some of the parents, the teaching of the tables should be begun a little earlier in the course.

The development of the ability to reason is one of the big results of the new course of study in arithmetic. Not long ago, hearing that a complaint had been made by the mother of a child in a 5−B room, regarding the teaching of arithmetic, I visited the room with the principal and tried to discover just what the youngsters could and could not do. I gave them several problems to test their ability to do mental arithmetic, and was surprised at the accuracy and speed with which they answered me. I then tried them on a problem which involved a little reasoning. I drew a picture of two faucets and of a pail placed beneath them. Stating that either one of the faucets could fill the pail alone in two minutes, I asked how long it would take to fill it if the two were running at the same time. Confidently expecting that the children would tell me four minutes, I was much gratified to receive the answer, one minute, from three−fourths of the class. I next changed the problem by stating that I would replace one of the faucets by a smaller one, which could fill the pail in four minutes. I then asked about how long it would take to fill the pail, if the two faucets ran together. A few told me three minutes, but the great majority guessed between one minute and two, the popular answer being about a minute and a half. I next asked what part of the pail would be filled at the end of one minute, and the children told me, without any difficulty, that it would be three−quarters full. My next question was, "How long exactly would it take, then, to fill the pail?" The first child that I called upon gave me the correct answer, one minute and twenty seconds. The principal expressed his astonishment and asked me to try the same problem on the eighth grade. I did so. These children, brought up under the old method of formal arithmetic, did not do nearly as well as did their younger brothers and sisters.

I have recently tried, in several parts of the city, a test involving five simple problems. Here it is:

1. Two boys start out together to race from Manchester to West Concord, a distance of 20 miles. One makes 4 miles an hour and the other 5 miles an hour. How long will it be before both have reached West Concord?
2. A man can row 4 miles an hour in still water. How long will it take him to row from Hill to Concord [24 miles one way] and back, if the river flows south at the rate of 2 miles an hour?
3. The same man again starts rowing from Hill to Concord in the spring when the water is high and the current is twice as swift as it was before. How long will it now take him to make the round trip?
4. Remus can eat a whole watermelon in 10 minutes. Rastus in 12. I suggest a race between them, giving each half of a melon. How long will it be before the melon is entirely gone?
5. The distance from Boston to Portland by water is 120 miles. Three steamers leave Boston, simultaneously, for Portland. One makes the trip in 10 hours, one in 12, and one in 15. How long will it be before all 3 reach Portland?

It looks easy enough, but I advise you to try it. I will guarantee that high school seniors, preparing for College Entrance Board Examinations in Mathematics, will not average 70 percent. I had some rather ridiculous results. I tried the fourth and fifth examples on a second grade the other day and had an almost perfect score, while a ninth−grade class in arithmetic, which had been taught under the old arithmetical curriculum, made a sorry showing. Out of twenty−nine in the class only six gave me the correct answer to problem five.

We have already seen results of our new course of study. The head of the English Department in our Central High−school [enrolling 2450 pupils] tells me that in the English classes made up of pupils who entered on February 1, 1935, there is a fluency and a readiness with the mother tongue that is surprising. The old−time diffidence is gone. Children are no longer tongue−tied and unable to put a new idea into words.

I am not surprised. I had expected a report like this. You will recall the terrible English used in one of our eighth grade rooms, taken down as it was spoken, which I have quoted in the first article. I went into the same room five years afterwards. The same teacher was in charge, and some of the children in the room were younger brothers and sisters of the previous group, but the methods of teaching had radically changed. With the stenographic report to the previous recitation in my hand, I asked this latter day group the same questions which I had propounded five years before to their older brothers and sisters. I pick out typical answers, and I assure you that I am not giving you the top of a "deaconed" barrel of apples.

> "When the numerators of any two fractions remain the same, the fraction with the smaller denominator is the largest."
>
> "The principle that we have proved is that the smaller the denominator gets − no, the larger the denominator gets, the smaller the fraction."
>
> "The larger the denominator is, the smaller the fraction would be if the numerator is the same."

> "The smaller the numerator gets, if the denominator remains the same, the smaller the fraction is."
>
> "The larger the denominator gets, the smaller the fraction will be, provided that the numerator remains the same."
>
> "The larger the denominator gets, provided the numerator remains the same, the smaller the fraction becomes."

I then tried an experiment which to me was the most conclusive of all. I read from the account in my hand typical answers which had been given in that same room five years before [of course they were not told that it was the same room] and these present day eighth graders shouted with laughter at statements which had not provoked a smile five years before. I asked them why they laughed and they proceeded to pick out the flaws in the reasoning and choice of words of their predecessors. To me it was the most heartening sign yet, and a prophecy of what we may expect when this present eighth grade shall have become seniors in our high schools.

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