1 Wetting your feet

1.1 *Estimate how many liters are in a barrel of oil and how many barrels of oil the United States imports every year.*

A: A barrel may be a few feet high, so $h \sim 1\,\mathrm{m}$, and have a diameter of $0.5\,\mathrm{m}$, so

$$V \sim 1\,\mathrm{m} \times 0.5\,\mathrm{m} \times 0.5\,\mathrm{m} \sim 0.25\,\mathrm{m}^3 = 250\,\ell.$$

The true value is $159\,\ell$. Perhaps one-half of the oil consumed in the United States is imported, and one-half of the oil consumed is for transportation, so I'll estimate the transportation amount and call it the imported amount. Maybe one-half of transportation is for passenger cars. There are probably two cars per family of four (and four per family in LA, zero or one per family in New York City), so roughly $1.5 \cdot 10^8$ cars. Each car drives maybe $1.5 \cdot 10^4\,\mathrm{mi\,yr^{-1}}$, and every $7.5\,\mathrm{mi}$ requires $1\,\ell$ of gasoline. The 7.5 was chosen so that the annual consumption per car is an easy $2000\,\ell$. If each barrel is $200\,\ell$, that's 10 barrels per year. So the total is $1.5 \cdot 10^9$ barrels/year for cars, or (doubling it) $3 \cdot 10^9$ barrels/year for all transportation and therefore for imports. According to the US Department of Energy `http://www.eia.doe.gov/emeu/international/petroleu.html`, the actual value is $1.21 \cdot 10^7$ barrels/day (in 2004) or $4.4 \cdot 10^9$ barrels/year.

1.2 *The mass of the sun is* $2 \cdot 10^{30}\,\mathrm{kg}$. *Estimate the mass of earth and the ratio*

$$\frac{\textit{mass of sun}}{\textit{mass of earth}}.$$

A: The earth is mostly made of rock, which has a density of roughly 5 or $6\,\mathrm{g\,cm^{-3}}$. Its radius is $6 \cdot 10^6\,\mathrm{m}$, so (using $4\pi/3 = 4$):

$$m \sim 6 \cdot 10^3\,\mathrm{kg\,m^{-3}} \times 4 \times (6 \cdot 10^6\,\mathrm{m})^3 \sim 5 \cdot 10^{24}\,\mathrm{kg}.$$

The true value is $6 \cdot 10^{24}\,\mathrm{kg}$. The ratio to the solar mass is

$$\frac{\text{mass of sun}}{\text{mass of earth}} \sim \frac{2 \cdot 10^{30}\,\mathrm{kg}}{5 \cdot 10^{24}\,\mathrm{kg}} \sim 4 \cdot 10^5.$$

2 Scaling

2.1 *How long is a year on Mars (*$d \sim 2.3 \cdot 10^{11}\,\mathrm{m}$ *from the sun)*

A: Don't compute it from scratch! Scale it against earth's value. Kepler's third law connects period and orbital radius:

$$T \propto r^{3/2}.$$

For earth, $r \sim 1.5 \cdot 10^{11}\,\mathrm{m}$, so Mars is 1.5 times farther from the sun than earth is. So its period T is $1.5^{3/2} \sim 1.9$ times greater, or $1.9\,\mathrm{yr}$. The true value is $1.88\,\mathrm{yr}$.

2.2 *Imagine a long steel rod hanging under its own weight. How does the maximum length before it breaks depend on its cross-sectional area?*

A: Let l be the maximum length and A the cross-sectional area. The weight supported by the rod is its own weight, or $\rho g A l$. The yield stress σ_{yield} is the maximum stress (a pressure) that steel can withstand before breaking. The stress is

$$\sigma \sim \frac{\text{weight}}{A} = \rho g l.$$

So when $\rho g l$ exceeds σ_{yield}, the rod breaks. This condition is independent of cross-sectional A.

3 Dimensional analysis

3.1 *What are the dimensions of power, energy flux, pressure, moment of inertia, and current, in terms of length L, mass M, time T, and charge Q?*

A:

power	$\mathrm{ML^2T^{-3}}$	energy/time
energy flux	$\mathrm{MT^{-3}}$	power/area
pressure	$\mathrm{ML^{-1}T^{-2}}$	force/area
moment of inertia	$\mathrm{ML^2}$	
current	$\mathrm{QT^{-1}}$	

3.2 *The volume V of a pyramid depends on its base area A and its height h. How many dimensionless groups do the three variables V, A, and h produce? Find them and write a general formula of the form*

$$\textit{group containing } V = f(\textit{other groups}).$$

A: The three variables, made of one dimension (length), produce two dimensionless groups. A pair could be V/Ah and h^2/A. Then

$$\frac{V}{Ah} = f\left(\frac{h^2}{A}\right).$$

3.3 *An object (ring, disc, sphere, etc.) rolls down an inclined plane without slipping. List the variables relevant to finding its acceleration a down the plane [Hint: a, I (its moment of inertia), and m (its mass) are three variables.] Find a set of dimensionless groups and then write the acceleration in the form*

$$\textit{group containing } a = f(\textit{other groups}).$$

If you double the radius of the object (e.g. use a big disc instead of a small disc), what happens to the acceleration?

A: Besides a, I, and m, the relevant variables include: g, since gravity causes the rolling; θ, since the angle of the plane affects the rolling speed; and r, the rolling radius of the object. These six variables and three dimensions make three groups. One is easy: θ is dimensionless already. Another is almost as easy: a and g are accelerations, so a/g is dimensionless. A third group uses the other three variables: I/mr^2. So

$$\frac{a}{g} = f\left(\theta, \frac{I}{mr^2}\right).$$

If r doubles, then I/mr^2 stays the same, so the acceleration cannot change. You can have fun guessing the form of f.

4 Drag

4.1 *Estimate the Reynolds number for a 747 and for a bumblebee.*

A: Re $\sim rv/\nu$ so for a 747:

$$\mathrm{Re} \sim \frac{30\,\mathrm{m} \times 300\,\mathrm{m\,s^{-1}}}{1.5 \cdot 10^{-5}\,\mathrm{m^2\,s^{-1}}} \sim 10^9.$$

The flow is highly turbulent. For a bumblebee:

$$\mathrm{Re} \sim \frac{1\,\mathrm{cm} \times 200\,\mathrm{cm\,s^{-1}}}{0.15\,\mathrm{cm^2\,s^{-1}}} \sim 10^3.$$

This flow is also turbulent.

4.2 *Estimate the drag force on you while bicycling at 15 mph and the mechanical power required. How can you check whether this power is reasonable?*

A: The drag force is

$$F \sim \rho v^2 A \sim 1\,\mathrm{kg\,m^{-3}} \times (7\,\mathrm{m\,s^{-1}})^2 \times 1\,\mathrm{m^2} \sim 50\,\mathrm{N}.$$

The power is Fv or about 350 W. To check this value, compare it to resting metabolism, which is 100 W. It's much harder than sitting around, but the number seems reasonable (though maybe a bit high, perhaps because the $c_\mathrm{d}/2$ factor was left out).

4.3 *Estimate the terminal velocity of a* 10 μm *fog droplet and the Reynolds number. How can you check whether your terminal velocity is reasonable? Is the flow viscous or turbulent?*

A: From the drag chapter,

$$v \sim \frac{2}{9}\frac{\rho_\mathrm{water}}{\rho_\mathrm{air}}\frac{gr^2}{\nu}.$$

Putting in the numbers and fudging the values slightly to make mental computation easy:

$$v \sim \frac{2}{10} \times 1000 \times \frac{10\,\mathrm{m\,s^{-2}} \times (10^{-5}\,\mathrm{m})^2}{2 \cdot 10^{-5}\,\mathrm{m^2\,s}} \sim 1\,\mathrm{cm\,s^{-1}}.$$

At this speed, a low cloud made of fog droplets, say at $h = 500\,\mathrm{m}$, would take $5 \cdot 10^4\,\mathrm{s} = 15\,\mathrm{hr}$ to settle. This time seems reasonable: overnight, fogs settle into valleys. The Reynolds number is

$$\mathrm{Re} \sim \frac{10^{-5}\,\mathrm{m} \times 10^{-2}\,\mathrm{m\,s^{-1}}}{\cdot 10^{-5}\,\mathrm{m^2\,s}} \sim 10^{-2}.$$

The flow is very viscous. For the fog droplet, the air is like a bath of honey.

5 Mechanical properties

5.1 *Estimate the energy to remove the final electron from* He$^+$*.*

A: He$^+$ has two protons and one electron, so its like hydrogen but with e^2 replaced by Ze^2 and $Z = 2$. The Bohr radius is

$$a_0 \sim \frac{\hbar^2}{m_e e^2},$$

so the equivalent radius in He$^+$ is a_0/Z. The energies scale as e^2/r, so replacing e^2 by Ze^2 and r by r/Z shows that the energies scale as Z^2 relative to hydrogen. For hydrogen, the ionization energy is 13.6 eV so here it is roughly 54 eV.

5.2 *Stiffness and strength both have dimensions of pressure. How do they differ in physical meaning?*

A: Stiffness is measured by applying a pressure and finding the fractional length change (the strain ϵ) that it produces. Strength is typically a small fraction of the stiffness: It is the maximum pressure that a material can withstand before it breaks. If the maximum strain is ϵ, then

$$\frac{\text{strength}}{\text{stiffness}} = \epsilon.$$

5.3 *How long can a steel wire be before it breaks under own weight? Does this calculation use stiffness or strength?*

A: From the earlier question, when $\rho g l$ exceeds σ_{yield} (the strength), the wire breaks. For steel, the stiffness is $2 \cdot 10^{11}\,\text{N}\,\text{m}^{-2}$ (Table 5.4 in the text) and $\epsilon_{\text{yield}} \sim 0.005$ (Table 5.6), so

$$\sigma_{\text{yield}} \sim 2 \cdot 10^{11}\,\text{N}\,\text{m}^{-2} \times 0.005 = 10^9\,\text{N}\,\text{m}^{-2}.$$

The density is $\rho \sim 7 \cdot 10^3\,\text{kg}\,\text{m}^{-3}$, so

$$l_{\max} \sim \frac{10^9\,\text{N}\,\text{m}^{-2}}{7 \cdot 10^3\,\text{kg}\,\text{m}^{-3} \times 10\,\text{m}\,\text{s}^{-2}} \sim 15\,\text{km}.$$

5.4 *Does the speed of sound in a material depend on its stiffness or on its strength?*

A: Sound waves are waves of compression or expansion, and sound propagation depends on how strongly the material resists compression or extension. So the speed of sound depends on stiffness.

6 Thermal properties

6.1 *Estimate the mean free path of air molecules at room temperature and $p = 0.1\,\mathrm{atm}$.*

A: The thermal chapter gives $\lambda \sim 10^{-7}\,\mathrm{m}$ as the mean free path for air at standard temperature and pressure. Reducing the pressure by a factor of 10 reduces the density by a factor of 10, but leaves other relevant parameters (thermal speed, molecular size) unchanged. From the thermal chapter, $n\lambda\sigma \sim 1$, where n is number density, λ is the mean free path, and σ is the molecular cross-section. Therefore reducing n by a factor of 10 increases λ by a factor of 10, to $10^{-6}\,\mathrm{m}$.

6.2 *What is the difference between κ (thermal diffusivity) and K (thermal conductivity), in dimensions and in physical meaning?*

A: Diffusivity has dimensions of $\mathrm{L}^2\mathrm{T}^{-1}$ and measures the speed at which heat can random walk in a material. Conductivity has dimensions of $\mathrm{MLT}^{-3}\Theta^{-1}$, where Θ stands for the dimensions of temperature. It depends on the diffusivity, but includes factors for: (1) how much heat is being carried by the molecules (or phonons), namely the specific heat per particle or phonon; and (2) the density of these carriers.

6.3 *Estimate the ratio*

$$\frac{\textit{time to cool a hardboiled egg with } r = 2.5\,\mathrm{cm}}{\textit{time to cool a hardboiled egg with } r = 2.0\,\mathrm{cm}}.$$

Does κ (diffusivity) or K (conductivity) determine the cooling times?

A: Cooking depends on heat diffusing to the center and cooking the proteins, and therefore depends on diffusivity. Diffusion is a random walk so $t \propto r^2$. Therefore

$$\frac{\text{time to cool a hardboiled egg with } r = 2.5\,\mathrm{cm}}{\text{time to cool a hardboiled egg with } r = 2.0\,\mathrm{cm}} \sim 1.25^2 \sim 1.6.$$

6.4 *Estimate the diffusion constant for a random walk with step size of $2\,\mathrm{m}$ and interval between steps of $10\,\mathrm{s}$.*

A: The velocity is $0.2\,\mathrm{m\,s^{-1}}$, so the diffusion constant is

$$D \sim \frac{1}{3} \times 0.2\,\mathrm{m\,s^{-1}} \times 2\,\mathrm{m} \sim 0.1\,\mathrm{m^2\,s^{-1}}.$$

The factor of 1/3 is magic, but the remainder is the only combination with the right dimensions.

6.5 *The diffusion constant for water molecules in water (i.e. for a 'green' water molecule random-walking among the other water molecules) is $D \sim 10^{-9}\,\mathrm{m^2\,s^{-1}}$. How long does a water molecule take, typically, to diffuse the length of an Olympic swimming pool?*

A: The time is

$$t \sim \frac{l^2}{D} \sim \frac{(50\,\mathrm{m})^2}{10^{-9}\,\mathrm{m^2\,s^{-1}}} \sim 3 \cdot 10^{12}\,\mathrm{s}.$$

Since one year is $\pi \cdot 10^7\,\mathrm{s}$, this time is $10^5\,\mathrm{yr}$! Water molecules, if they ever cross the pool, don't do it by diffusion.