Solutions to Problem Set 7

Problem 1. *Heat Loss By Swimmers*

Do world class distance swimmers overheat? Note that the Prandtl number of water, $Pr \equiv \nu/\kappa \approx 6$.

The heat generated in sprints is $\sim 3\,\mathrm{kW}$ (see the *Human Physical Performance* handout). The mechanical power is $\sim$ 500–700 W but the heat generated is 4 times that because muscles are only so efficient. Probably endurance athletes, who use aerobic muscles, generate somewhat less, say 1.5 kW. The factor of 2 is reasonable: sprinters on land can run a 100 m dash in $\sim 10\,\mathrm{s}$, for a $v \sim 10\,\mathrm{m/s}$. The equivalent milestone in distance runs is the '4-minute mile' (first run by the neurophysiologist Roger Bannister), for $v \sim 1600\,\mathrm{m}/240\,\mathrm{s} \sim 7\,\mathrm{m/s}$. Since athletes here are mostly doing work against turbulent v^2 drag, the ratio of power outputs is $\sim 0.7^2 \sim 0.5$.

From the *Human Performance* handout, the 1500 m freestyle record is 15 min $\sim$ 1000 s, so $v \sim 150\,\mathrm{cm/s}$. The viscous boundary layer thickness is set by how far momentum can diffuse while the fluid travels along the swimmer. So $\delta_\nu \sim \sqrt{\nu l/v}$, where l is the swimmer's length. At first sight, one would expect the thermal boundary layer to scale similarly, but with the momentum diffusivity, ν, replaced by the thermal diffusivity, κ. So $\delta_t \sim \sqrt{\kappa l/v}$. This would be true in an ideal gas, where the Prandtl number is 1 ($\nu = \kappa$). But for water, $\mathrm{Pr} > 1$, which means that the thermal boundary layer lies inside the viscous boundary layer. At the edge of the thermal boundary layer, the fluid velocity is not the full free-stream velocity, v, which means the time available for thermal diffusion is a bit longer than l/v. So the thermal boundary layer is actually a little farther out than our first guess, $\sqrt{\nu l/v}$.

To work it out somewhat quantitatively, we first choose a sensible unit system so we don't carry useless constants around in the derivation. Gross lengths will be scaled to l, velocities to the free stream value, and the viscosity to ν. In these units, the viscous boundary layer thickness is 1 (though not l!), and $\kappa \equiv \mathrm{Pr}^{-1}$. We assume that the fluid velocity in the viscous boundary layer at height y is $\sim y^\alpha$, where α is some exponent. Let δ be the (scaled) thermal boundary layer thickness. The velocity at that height is δ^α. So the time for heat to diffuse is $\tau \sim \delta^{-\alpha}$, and the heat can diffuse out to $\delta \sim \left(\mathrm{Pr}^{-1} \times \tau\right)^{1/2} \sim \left(\mathrm{Pr}^{-1}\delta^{-\alpha}\right)^{1/2}$. Solving, $\delta \sim \mathrm{Pr}^{-1/(2+\alpha)}$. The simplest choice of velocity scale is $\alpha = 1$; this also agrees well with experiment. Then $\delta \sim \mathrm{Pr}^{-1/3}$. Scaling back to normal units,

$$\delta_t \sim \mathrm{Pr}^{-1/3}\delta_\nu.$$

To determine δ_ν, we take the length scale to be the swimmer's height, $l \sim 200\,\mathrm{cm}$. Plugging in the numbers, the viscous boundary layer has thickness $\delta_\nu \sim (10^{-2} \times 200/150)^{1/2} \sim 0.1\,\mathrm{cm}$. The thermal boundary layer therefore has thickness $\delta_t \sim 6^{-1/3} \times 0.1 \sim 0.075\,\mathrm{cm}$, not a large difference from δ_ν. We'll just use the 0.1 cm value, and maybe increase our power flux at the end by 30%.

The outward heat flux is

$$\mathcal{P}_{\mathrm{cool}} \sim \rho c_v \kappa \frac{\Delta T}{\delta_t},$$

where ΔT is the temperature difference between the swimmer and distant water, and c_v is the specific heat of water. A swimmer is at 37°C, and the water is say at 20°C, so $\Delta T \sim 20°\mathrm{C}$. As always, $c_v \sim 1\,\mathrm{cal\,g^{-1}\,K^{-1}} \sim 4\cdot 10^7\,\mathrm{erg\,g^{-1}\,K^{-1}}$. For later use, $K \equiv \rho c_v \kappa \sim 1 \times 7 \times 1.5\cdot 10^{-3} \sim 6\cdot 10^4\,\mathrm{erg\,cm^{-1}\,s^{-1}\,K^{-1}}$. Putting in numbers,

$$\mathcal{P}_{\mathrm{cool}} \sim 1 \times \underbrace{4\cdot 10^7}_{c_v} \times \underbrace{1.5\cdot 10^{-3}}_{\kappa} \times \underbrace{\frac{20}{0.1}}_{\Delta T/\delta_t} \sim 10^7\,\mathrm{erg\,cm^{-2}\,s^{-1}} \sim 1\,\mathrm{W/cm^2}.$$

Suppose a swimmer is a cylinder of radius $R \sim 12\,\mathrm{cm}$. Then $A \sim 2\pi Rl \sim 6 \times 12 \times 200 = 1.5 \cdot 10^4\,\mathrm{cm}^2$. The available cooling power is $\mathcal{P}_{\mathrm{cool}}A$, so

$$P_{\mathrm{cool}} \sim 1\,\mathrm{W/cm}^2 \times 1.5 \cdot 10^4\,\mathrm{cm}^2 \sim 15\,\mathrm{kW}.$$

15 kW is much more than the 1 kW or 2 kW generated, so the swimmer won't overheat (and we won't bother to correct it by 30%, since it's plenty already).

Problem 2. *Boundary Layer Drag*

Consider the drag on a sphere moving through a homogeneous fluid. What fraction of the total drag is contributed by friction associated with the boundary layer? Assume that the Reynolds number is in the regime, $1 \ll Re \leq 10^5$, such that the boundary layer is laminar but the wake is turbulent.

Simple method. For $1 \ll \mathrm{Re} \leq 10^5$, the boundary layer is laminar but the wake is turbulent. From the second lecture, the drag force is

$$F_d \sim (c_d/2)\rho v^2 A, \qquad \text{where } c_d \sim \begin{cases} 1/\mathrm{Re}_\delta & \text{for the laminar boundary layer,} \\ 1 & \text{for the turbulent wake.} \end{cases}$$

Here Re_δ is the Reynolds number in the boundary layer. Boundary layer drag is the skin friction; the turbulent wake produces the form (or pressure) drag. So the ratio of these drag forces is $\alpha \equiv F_{\mathrm{skin}}/F_{\mathrm{form}} \sim \mathrm{Re}_\delta^{-1}$. From the 17 April lecture, we know Re_δ is $\sim \mathrm{Re}^{1/2}$. So

$$\frac{F_{\mathrm{skin}}}{F_{\mathrm{form}}} \sim \mathrm{Re}^{-1/2} \sim \sqrt{\frac{\nu}{Rv}}. \tag{1}$$

Second method. A sphere of radius R moves with velocity v through the fluid of kinematic viscosity ν. The boundary layer thickness, δ, is set by how far momentum can diffuse in the time it takes fluid to cross the sphere. So

$$\delta \sim \sqrt{\nu R/v}. \tag{2}$$

In the rest frame of the sphere, the velocity goes from zero at the surface of the sphere to v outside the boundary layer. This velocity gradient, v/δ, creates a viscous stress, $T \sim \rho\nu(v/\delta)$. The stress acting over an area $\sim R^2$ produces the skin friction drag force $F \sim R^2 T \sim R^2\rho\nu v/\delta$. Putting in the boundary layer thickness from (2),

$$F_{\mathrm{skin}} \sim \rho\nu^{1/2}(vR)^{3/2}.$$

The form drag is the usual

$$F_{\mathrm{form}} \sim \frac{c_d}{2}\rho A v^2 \sim\sim \frac{1}{4}\rho\pi R^2 v^2 \sim \rho v^2 R^2, \tag{3}$$

where we have taken $c_d \sim 0.5$ for a sphere. Their ratio is

$$\alpha = \frac{F_{\mathrm{skin}}}{F_{\mathrm{form}}} \sim \sqrt{\frac{\nu}{vR}}, \tag{4}$$

which agrees with (1). For a fly buzzing about,

$$R \sim 0.5\,\mathrm{cm}, \qquad v \sim 100\,\mathrm{cm/s} \qquad \text{and} \qquad \nu \sim 0.2\,\mathrm{cm}^2/\mathrm{s}.$$

So $\mathrm{Re} \sim Rv/\nu \sim 250$ and $\alpha \sim \mathrm{Re}^{-1/2} \sim 0.07$. Skin friction is not very important unless the Reynolds numbers are close to 1. But in a streamlined wing, which has a small frontal area, the skin friction might be relatively more important because the form drag depends mostly on the frontal area, while the skin friction depends more on the total surface area.

Problem 3.

A freight train moving at 45mph sees a car on the tracks ahead, and locks its brakes.

a) Estimate the stopping distance (the coefficient of friction for steel sliding on steel is 0.4).

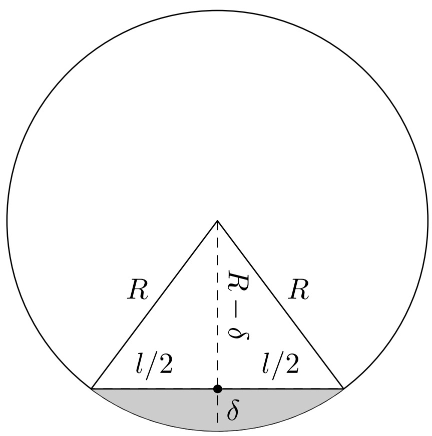
b) Estimate the contact area between each wheel and the rails.

c) Estimate the depth to which heat diffuses during the time the contact area of each wheel takes to slide its own length.

d) Thus derive an approximate equation for the peak surface temperature of the rails, and estimate its value. Is melting (lubrication!) a problem?

e) For how long after the train stops will the rails remain warm to the touch?*

a) The normal force is $N = mg$, where m is the mass of the train. So the frictional force is $F_\mu = \mu N = \mu mg$, where we are given the coefficient of friction, $\mu = 0.4$. The resulting deceleration is $a = F/m = \mu g = 400\,\mathrm{cm/s^2}$. In cgs units, 45 mph is 2000 cm/s, so the stopping distance is $v^2/2a = 4 \cdot 10^6/800 = 5000\,\mathrm{cm}$, or half a football field.



b) Imagine each wheel is a cylinder of thickness w and radius R. The bottom part of the wheel gets flattened by the weight of train it holds up (see the preceding figure). In the figure, the flattened section is shaded. Let the maximum flattening be δ. The average strain is then $\epsilon \sim \delta/2R$, and this strain produces a force $F \sim AB\epsilon$, where A is the area of contact and B the bulk modulus of steel. The flat section has length along the track of $l \sim 2\sqrt{2\delta R}$. Substituting in $\delta \sim 2\epsilon R$, the length is $l \sim 4R\epsilon^{1/2}$, and the area of contact is

$$A \sim lw \sim 4Rw\epsilon^{1/2}. \tag{1}$$

The upward force is then

$$F \sim AB\epsilon \sim 4BRw\epsilon^{3/2}. \tag{2}$$

Each freight car has 3 pairs of wheels in the front and 3 pairs in the back, so say 10 wheels. A car say has dimensions $10\,\mathrm{m} \times 2\,\mathrm{m} \times 2\,\mathrm{m} = 40\,\mathrm{m}^3$. If it's filled with water, that would be 40 tons, so let's say $m_{\mathrm{car}} \sim 50$ tons or $5 \cdot 10^7$ g. Each wheel supports $mg/10$ or $F \sim 5 \cdot 10^9$ dyn. Let's say $R \sim 25$ cm and $w \sim 8$ cm. For steel, $B \sim 2 \cdot 10^{12}\,\mathrm{dyn/cm^2}$. Then from (2),

$$\epsilon \sim \left(\frac{F}{4BRw}\right)^{2/3} \sim \left(\frac{5 \cdot 10^9}{4 \times 2 \cdot 10^{12} \times 25 \times 8}\right)^{2/3} \sim 2 \cdot 10^{-4}.$$

* *We will not post bail for students attempting to determine the answer to parts (a) and (e) by parking their cars on a railway level crossing and waiting upstream!*

This provides a good safety margin of the yield strain, $\sim 10^{-2}$. From (1), the contact area is

$$A \sim 4 \times 25 \times 8 \times \sqrt{2 \cdot 10^{-4}} \sim 12\,\mathrm{cm}^2.$$

c) From part b) above, the contact area has length $l = A/w \sim 1.5\,\mathrm{cm}$. We'll take the velocity to be $v_{\max} \sim 2000\,\mathrm{cm/s}$, so the time it takes the contact area to slide over l is $\tau \sim 0.7\,\mathrm{ms}$. For metals $\kappa \sim 10^2\,\mathrm{cm}^2/\mathrm{s}$, so heat will diffuse to a depth $h \sim \sqrt{\kappa\tau} \sim 0.25\,\mathrm{cm}$.

d) The power dissipated through the contact area is $P \sim \mu F v$, where F is the weight a wheel supports. So the power flux delivered through the contact area is $\mathcal{P} \sim P/A$. This flux is approximately the flux conducted inwards, $\mathcal{P}_{\mathrm{in}} \sim K\Delta T/\delta$. Equating the power fluxes,

$$\Delta T \sim \frac{\mu F v \delta}{AK}.$$

Since $\delta \propto v^{-1/2}$, the temperature difference is $\propto v^{1/2}$. We are interested in the peak surface temperature, so we use the largest velocity, $v \sim 2000\,\mathrm{cm/s}$ (at the beginning of the braking). From part c), the thermal boundary layer thickness is $\delta \sim 0.25\,\mathrm{cm}$. From the materials sheet, K for iron is $\sim 0.2\,\mathrm{cal\,cm^{-1}\,s^{-1}\,K^{-1}} \sim 8{\cdot}10^6\,\mathrm{erg\,cm^{-1}\,s^{-1}\,K^{-1}}$. From part b), each wheel supports $F \sim 5 \cdot 10^9\,\mathrm{dyn}$. Putting all these in,

$$\Delta T \sim \frac{0.4 \times 5 \cdot 10^9 \times 2000 \times 0.3}{12 \times 8 \cdot 10^6} \sim 10^4\,\mathrm{K}!$$

Melting is quite a problem. Except that the pressure is $\sim 2500\,\mathrm{atm}$, which might keep the steel somewhat solid.

e) Pretty quickly the heat fills the cross-section of the rail—the 0.25 cm thick boundary layer diffuses into the full height of the rails (say $h \sim 10\,\mathrm{cm}$). The average temperature in the thermal boundary layer is $\sim \Delta T/2$; the diffusion cuts that average temperature by a factor of $\sim 10/0.25 = 40$, say to $100°\mathrm{C}$. How long does it take the heat to travel this 10 cm? The heat is diffused by the electrons, which have the higher κ of $10^2\,\mathrm{cm}^2/\mathrm{s}$. So going 10 cm takes $\sim 1\,\mathrm{s}$. At very high temperatures, κ is quite a bit smaller, $\kappa \sim T^{-1}$, but even so, the time is only a few seconds.

Getting the heat out of the rails will take much longer. We'll assume convective cooling—the hot rails generate air currents of speed v which transport heat away. The heat flux from the rail will be worked out just as in the swimming problem, except here the flux cools the rails, whereas the swimmer keeps burning more fuel and generating more heat. The viscous and thermal boundary layers ($\mathrm{Pr} = 1$ for air) have thickness $\delta \sim \sqrt{\nu h/v}$. Taking $v \sim 10\,\mathrm{cm/s}$, we find $\delta \sim 0.2 \times 10/10 \sim 0.4\,\mathrm{cm}$.

The heat flux across this layer is $F \sim K_{\mathrm{air}}\Delta T/\delta$, where ΔT is the temperature excess the rails have over atmospheric temperature. This flux cools the track (it's assumed from out the top for now). F/h is the heat loss per volume, so the cooling rate is

$$d(\Delta T)/dt \sim \frac{F}{h\rho c_v},$$

where ρc_v is the volume heat capacity of the track. Putting in the expression for F,

$$\frac{d(\Delta T)/dt}{\Delta T} \sim \frac{K_{\mathrm{air}}}{\delta h \rho c_v}.$$

Air is a very good insulator, so we'll take $K_a \sim 10^{-4}\,\mathrm{cal\,cm^{-1}\,s^{-1}\,K^{-1}}$. From the materials sheet, $\rho c_v \sim 0.3\,\mathrm{cal\,cm^{-3}\,K^{-1}}$, and from above, $\delta \sim 0.4\,\mathrm{cm}$. Putting the numbers in,

$$\frac{d(\Delta T)/dt}{\Delta T} \sim \frac{10^{-4}}{0.4 \times 10 \times 0.3} \sim 10^{-4}\,\mathrm{s}^{-1}.$$

So the time constant is $\tau \sim 10^4\,\mathrm{s}$. Since the track has three sides exposed to air, perhaps we take one-third of that, and say the track cools with time constant ~ 1 hour. After a couple of time constants, the $100°\mathrm{C}$ excess should be cut down to $\sim 10°\mathrm{C}$, which isn't very warm.

Problem 4. *Skating.*

The coefficient of steel sliding on ice at temperatures between $-11°$C and $-5°$C is about 0.005.

a) *Estimate the ratio of power a speed skater uses to overcome sliding friction compared to the power to overcome wind resistance (the world records in 5km and 10km speed skating are held by Koss, respectively $6^{\mathrm{m}}35^{\mathrm{s}}$ and $13^{\mathrm{m}}30^{\mathrm{s}}$).*

b) *Estimate the forward force that must be applied by the skating strokes to maintain speed against the total drag. You should find that this is large compared to the sliding friction, but small compared to the body weight. How is speed skating possible if the forward force must be large compared to the sliding friction (banana peel effect)? Are wind resistance and sliding friction the only relevant dissipation?*

a) The friction force is $F_\mu \sim \mu mg$, so the power expended due to friction is

$$P \sim F_\mu v \sim \mu mgv.$$

Koss skates with velocity $v \sim 10\,\mathrm{km}/13.5\,\mathrm{min} \sim 10^6\,\mathrm{cm}/800\,\mathrm{s} \sim 1200\,\mathrm{cm/s}$, and probably has $m \sim 10^5$ g. Using $\mu = 0.005$, the power is

$$P_\mu \sim 0.005 \times 10^5 \times 10^3 \times 1200 \sim 6 \cdot 10^8\,\mathrm{erg/s} = 60\,\mathrm{W}.$$

The power expended against wind resistance is the usual $P = F_d v \sim (c_d/2)\rho v^3 A$, where A is the frontal cross-section. For a traveling cylinder, $c_d \sim 1$. Let's say that A is one-third the total body area of $1.5 \cdot 10^4\,\mathrm{cm}^2$ (this was calculated in problem 2); then $A \sim 5 \cdot 10^3\,\mathrm{cm}^2$. This is also the frontal area of cyclists given in the *Human Performance* handout. Using $\rho \sim 10^{-3}$,

$$P = 0.5 \times 10^{-3} \times (1200)^3 \times 5 \cdot 10^3 \sim 4 \cdot 10^9\,\mathrm{erg/s} = 400\,\mathrm{W}.$$

The ratio $P_\mu/P \sim 0.15$.

b) The total drag power is $P \sim F_{\mathrm{tot}} v \sim 5 \cdot 10^9\,\mathrm{erg/s}$. From above, $v \sim 1200\,\mathrm{cm/s}$, so $F_{\mathrm{tot}} \sim 4 \cdot 10^6$ dyn. The skater's mass is $\sim 10^5$ g, so the weight is $\sim 10^8$ dyn, which is larger than F_{tot} by a factor of 25. However, the sliding friction is $F_\mu \sim \mu mg \sim 0.005 \times 10^8\,\mathrm{dyn} = 5 \cdot 10^5\,\mathrm{dyn}$; this is one-tenth of F_{tot}. No one ever accelerated to racing speeds without digging their skates into the ice and pushing off, using a large coefficient of static friction.

Another dissipation is the form drag of the blade in the melted ice it travels through. The Reynolds number is $\mathrm{Re} \sim Rv/\nu \sim 0.1 \times 1000/0.01 \sim 10^4$. The wake is turbulent, but the boundary layer is laminar: $c_d \sim 1$. So $P_d \sim \rho A v^3/2$. Forgetting the 2, and taking the cross section of blade in the water to be $0.1\,\mathrm{cm} \times 0.1\,\mathrm{cm}$, $P_d \sim 1 \times 10^{-2} \times 10^9 \sim 1\,\mathrm{W}$. This is negligible compared with the 500 W of air drag.

Problem 5. *More skating.*

The origin of the low coefficient of skating friction on ice is poorly understood. Decide whether you think two explanations commonly given are likely or not:

a) *Pressure melting. Since ice is less dense than water, pressure reduces the melting temperature. Estimate the change in melting temperature under the pressure of a skate supporting a human. Do you think ice at -11C will liquefy?*

b) *Frictional heat melting. Use your results of problem 3 to estimate the change in surface temperature of the ice during skating. Could that melt ice at $-11°$C?*

a) For water, the cohesion energy is $\epsilon_c \sim 0.5\,\mathrm{eV}$ per molecule, and the melting temperature is $T_{\mathrm{melt}} = 273\,\mathrm{K}$, so $kT_{\mathrm{melt}} = 0.023\,\mathrm{eV}$. Their ratio is $\epsilon_c/kT_{\mathrm{melt}} \sim 21$. The pressure P will alter the effective cohesion energy density by P, so it will alter the melting temperature by $P/21$. High pressures favor the denser phase, which means the melting temperature will drop.

We'll work out the ΔT for $P = 1000\,\mathrm{atm}$, since we can check that value in my table; ΔT for other pressures will just be linear in P. Taking $3\,\text{Å}$ for the molecular spacing,

$$1000\,\mathrm{atm} \times \frac{10^6\,\mathrm{erg/cm}^3}{1\,\mathrm{atm}} \times \frac{(3 \cdot 10^{-8}\,\mathrm{cm})^3}{1\,\mathrm{molecule}} \times \frac{1\,\mathrm{eV}}{1.6 \cdot 10^{-12}\,\mathrm{erg}} \sim 0.017\,\mathrm{eV/molecule}.$$

This is $0.74kT_{\mathrm{melt}}$, so $1000\,\mathrm{atm} \sim 0.74 \times 273\,\mathrm{K} \sim 200\,\mathrm{K}$. The change in melting temperature will therefore be $200/21 \approx 9.6°\mathrm{C}$. A handbook (*Smithsonian Physical Tables*) gives $8.8°\mathrm{C}$. For this class call $\Delta T \sim 10°\mathrm{C}$ for $1000\,\mathrm{atm}$, so

$$\Delta T \sim \left(\frac{P}{100\,\mathrm{atm}}\right) °\mathrm{C}.$$

A human masses say $10^5\,\mathrm{g}$, and each blade may have cross-section $\sigma \sim 0.1\,\mathrm{cm} \times 10\,\mathrm{cm} \sim 1\,\mathrm{cm}^2$. So the pressure is $mg/\sigma \sim 10^8\,\mathrm{dyn/cm}^2$, which is $100\,\mathrm{atm}$. Maybe it's half that if you're on two blades, but that'll turn out irrelevant. The melting temperature falls by only $1°\mathrm{C}$, not enough to melt $-11°\mathrm{C}$ ice.

b) A blade has length $l \sim 10\,\mathrm{cm}$, and it moves along at $1000\,\mathrm{cm/s}$, so the frictional heating has a time $\tau \sim 10\,\mathrm{ms}$ to diffuse into the ice. So the thermal boundary layer has thickness $\delta_t \sim \sqrt{\kappa\tau} \sim \sqrt{1.5 \cdot 10^{-3} \times 10^{-2}} \sim 4 \cdot 10^{-3}\,\mathrm{cm}$. From problem 3d),

$$\Delta T = \frac{\mu F v \delta}{AK}.$$

Here $\mu = 0.005$, $F/A \sim 10^8\,\mathrm{dyn/cm}^2$, $v \sim 10^3\,\mathrm{cm/s}$, $\delta \sim 4 \cdot 10^{-3}\,\mathrm{cm}$ and (from problem 1) $K_{\mathrm{water}} \sim 6 \cdot 10^4\,\mathrm{erg\,cm^{-1}\,s^{-1}\,K^{-1}}$. Putting all this in,

$$\Delta T \sim \frac{0.005 \times 10^8 \times 10^3 \times 4 \cdot 10^{-3}}{6 \cdot 10^4} \sim 30°\mathrm{C}.$$

Since $L_{\mathrm{fus}} \sim 80\,\mathrm{cal/g}$, melting the ice requires an effective temperature 'rise' of $L_{\mathrm{fus}}/c_v \sim 80°\mathrm{C}$. This $30°\mathrm{C}$ rise from friction is enough to heat up the ice from $-11°\mathrm{C}$ to $-1°\mathrm{C}$, but it's not enough to melt it.

But let's imagine that the ice doesn't melt. The coefficient of friction between ice and steel is ~ 0.2 (this may not be right, but it's certainly much higher than 0.005). Increasing μ by a factor of 40 will make $\Delta T \sim 40 \times 30°\mathrm{C} \sim 1200°\mathrm{C}$, more than enough to melt the ice. And this ΔT doesn't consider that the pressure may be concentrated at the front of the blade, making F/A larger by say a factor of 5. As the ice melts, μ drops back, reducing the heating. This negative feedback loop probably keeps a thin layer of water under the blade barely melted.

Another effect is that with a phase change, the thermal diffusion changes. So most of the heat, instead of diffusing down to $4 \cdot 10^{-3}\,\mathrm{cm}$, will go into melting a thinner layer, say of thickness $\delta/4 \sim \cdot 10^{-3}\,\mathrm{cm}$. The heat in such a thin layer would be sufficient to melt the ice.

In short, frictional heating explains ice skating much better than pressure-melting. As one student pointed in section, if pressure were melting the ice, when you took your blade off the ice, the water would re-freeze. But you don't find ice crystals on the bottom of the skate blade; instead the blade is wet.