Physics 103c

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Order of Magnitude Physics

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Problem Set 6

Due at *beginning* of class *Wednesday, 17 May 1995*

Homework Problems:

1. Ship Wakes
   a) Prove that surface gravity waves generated by a ship in steady motion are confined to a wedge with opening angle $2\theta = 2\sin^{-1}(1/3)$.
   b) How does the wavelength of forward propagating waves depend upon the speed of the ship?

2. River Rapids

   Two conditions must be satisfied for the formation of rapids. The Froude number of the flow must exceed unity, and the river bottom must have boulders of size comparable to the river depth. Assume that the boulders are sparse so that their presence does not affect the mean flow.

   a) What is the limiting downstream slope required to produce rapids? Derive an analytic expression and then evaluate it numerically.

3. Atmospheric Scintillation

   The atmosphere contains large cells of gas at very different temperatures separated by shear layers (one obvious place is near the ground, another is in convection cells). The shear is turbulent, so the turbulence mixes the hot and cold air down to very fine scales. The resulting complicated temperature distribution has a roughly Kolmogorov spectrum, such that two parcels of air separated by distance ℓ have root-mean-square temperature differences of order

$$\langle \delta T^2/T^2 \rangle^{1/2} = 5 \times 10^{-5} (\ell/1\,\mathrm{cm})^{1/3}\,.$$

   a) Explain why stars (angular size $< 10^{-7}$ radian) twinkle, but planets (angular size $\sim 5 \times 10^{-5}$ radian) don't.
   b) Estimate the timescale on which stars twinkle.

   Note: the index of refraction of air at optical wavelengths satisfies $n - 1 \approx 3 \times 10^{-4}$.

4. Southern California must find new supplies of fresh water. To be economically feasible the cost must be competitive with the current Pasadena price of 20 cents per 10^2 gallons. Use \$1.25 per gallon of gasoline as the cost of energy. In b) and c) below the total rate of supply enters into the unit cost. Assume that the supply is intended to meet the needs of 20 million people.

   a) **Towing icebergs from the arctic.** What size iceberg (assume it is a cube) would supply the needs of 20 million people for one year? How fast could we afford to tow it? Would melting in transit be a serious problem? At fixed speed, is it cheaper to tow one large iceberg or many smaller ones?

   b) **Pumping water over level ground from large rivers that empty into the Pacific Ocean near the border between Canada and the United States.** To keep the costs of pumped water comparable with that of current supplies, how large a diameter pipe would we have to build?

5. Water gets from the roots of trees to their leaves through the long hollow cells of the xylem. The radii of the xylem cells are $\sim 20\,\mu$m, and the pores in the leaves have radii ~ 50Å.

   a) By arguments similar to those given in class for solids, estimate the tensile strength of pure water, with no dissolved gasses (the units are those of pressure, so the tensile strength is also called the cavitation pressure).

   b) One limit to the height of a tree is set by the capillary pressure defect of water in the pores (show that the capillary rise in the xylem is insignificant: water doesn't pour out of the stumps of cut trees). Another limit is set by the tensile strength of the column of water joining the root to the leaf pores. How close are the tallest trees to these limits?

   c) Estimate the velocity of flow of water up a tree trunk. What additional pressure drop (beyond the hydrostatic one) between the roots and the leaves are required to maintain this velocity of flow in the xylem?