Physics 103c

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Order of Magnitude Physics

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Problem Set 8

Due at *beginning* of class *Wednesday, 31 May 1995*

Homework Problems:

1. Generation of Sound by Turbulence
   a) Consider three-dimensional fluid turbulence with characteristic velocity v, outer scale L, and Mach number $M \sim v/c_s \ll 1$. What is the approximate amplitude of the turbulent pressure fluctuations?
   b) Estimate the efficiency of acoustic radiation by the turbulence. Express the power radiated per unit volume as a fraction of the total energy dissipation rate per unit volume, namely $\epsilon \sim \rho v^3/L$.

      Hint: Quadrupoles are the lowest order acoustic multipoles.

2. Information Transfer Rates
   a) At what bit rate do you absorb information when reading a novel?
   b) How long would it take to transmit over a video channel, bandwidth $\Delta\nu \approx 4 \times 10^6$ cycles per second, the information contained in the human genome, approximately one meter of DNA?

3. Information Content
   a) Estimate the number of distinct words that you can recognize. Explain your methodology.
   b) How many distinct words would you estimate there are in the collected works of Shakespeare?

4. Neglecting the small bell at the end, a clarinet with all the finger holes covered can be approximated by a cylinder $L = 65$cm long and $2a = 1.5$cm in interior diameter, driven by periodic pressure pulses at the mouth end. Consider frequencies close to the fundamental frequency (derived and illustrated in class). Your answers should not involve any quantities other than L, a, and physical properties of air (sound speed, thermal and viscous diffusivities).
   a) Estimate the Q (π times the number of wave cycles required to reduce the amplitude by $1/e$, or 2π times the number of wave cycles required reduce the power by $1/e$) of the clarinet. Be sure you consider heat transfer and viscous losses as well as radiative losses. Which dominate?
   b) Neglecting the bell at the end, estimate the radiative efficiency [ratio of acoustic power out to power driving the pressure pulses at the mouth] of the cylindrical clarinet. How would this change if the diameter of the tube were doubled? What is the effect of the small flared bell at the end of a real clarinet?

5. The upper 3/4 of piano strings are bare steel wires, stretched to the yield point of steel.
   a) Estimate the speed of transverse waves on such a piano string, and compare to the speed of sound in air.
   b) Estimate (using only the properties of steel) the length of a piano string whose fundamental frequency ν_1 is middle C (262Hz).
   c) The middle C piano string is about $d = 0.12$cm in diameter. The restoring force on a bent string has contributions from the differential stresses on its two sides ('stiffness' cf. the vibrating or buckling strut, present even if the center of the string is not in tension) and from the tension. Show that although the frequencies of the modes of a string of zero diameter are harmonically related (integer multiples n of the fundamental frequency ν_1), the stiffness term introduces an anharmonic term: $\nu_n = n\nu_1(1 + An^2)$, and estimate its coefficient A. By what percentage is the 4th harmonic of middle C sharp? Does the problem get better or worse for higher C notes? This effect means that beats will always be heard between the upper harmonics when octave intervals are played on a piano, no matter how it is tuned (to minimize the effect, piano tuners actually 'stretch' the tuning of octave strings, so their fundamental frequencies are not exactly a factor of 2 apart, but their overtones are more consonant).

6. Invent a problem of your own.