Solutions to Problem Set 5

1. How fast can grass grow? Consider both wet and dry climates.

If there is no limit to the supply of water, ground based nutrients and carbon dioxide then the limiting factor is the availability of sunlight. The solar constant outside the atmosphere is 1.4×10^6ergs cm^{-2}s^{-1}. Between 60 and 85 percent of the total solar radiation is transmitted by the atmosphere, depending upon the airmass (angle of sun on sky) and the water vapor content of the atmosphere (see Allen page 131). Assuming that 65 percent on average of the sunlight is transmitted by the atmosphere and that the grass will receive sunlight for 10 hours per day then the average power S available from sunlight is $S \sim 4 \times 10^5$ergs cm^{-2}s^{-1}. Burning plant material releases about $h = 4 \times 10^{11}$ ergs/gram. Burning is converting cellulose plus oxygen to CO_2 and H_2O. When the plant grows, it must therefore use at least an equal amount of energy from sunlight to drive the inverse reaction (H_2O and CO_2 converted into sugars and cellulose). Hence the growth rate of grass, if it uses sunlight with efficiency ϵ is

$$\frac{dm}{dt\, dA} = \frac{\epsilon S}{h} = 0.09\epsilon \,\mathrm{g\ cm^{-2} day^{-1}} = 30\epsilon \,\mathrm{g\ cm^{-2} yr^{-1}}\,. \tag{1}$$

This tells how many tons of grass clippings you have to haul away. To estimate the rate at which the grass lengthens, we need to know the mean density of a grass patch. Grass blades have density $\rho_{\rm grass} \sim 1$g cm^{-3}, but they fill only $f \sim 10^{-2}$ of the volume of a lawn (the evolutionary reason can be understood by noting that the grass blades in front of Millikan have $twh = 0.02$cm $\times$ 0.2cm $\times$ 10cm, so a volume 4×10^{-2} cm^3. But, averaged over a day, each blade shadows a mean volume $\simeq (1/2)h^2(1/3)w = 3$ cm^3 where no other grass can get sunlight. Hence the filling factor should be $f \sim 4 \times 10^{-2}$ cm^3/3cm^3 $\sim 10^{-2}$, as observed). The growth rate, $\frac{dh}{dt}$, is thus

$$\frac{dh}{dt} \sim \frac{dm}{dt\, dA}\frac{1}{f\rho_{\rm grass}} \sim 2\text{cm per day}\left(\frac{\epsilon}{0.2}\right)\left(\frac{0.01}{f}\right)$$

Since grass has to grow roots as well as blades, and the roots have mass at least comparable to the blades', probably less than half of this dh/dt will appear as above-ground growth. With $\epsilon = 0.2$, this gives $dh/dt \sim 1$ cm day^{-1}, about what is observed in a well-watered Pasadena lawn in summer.

What about water? A typical long carbohydrate is $(C_6H_{10}O_5)^n$, so about five percent of the plant's mass is hydrogen. Hydrogen must come from the water, and water is 10 percent hydrogen by mass. Hence to grow a gram of plant material requires ~ 0.5 g of water. In recent years, Southern California's precipitation has been only ~ 15 cm y^{-1}. Comparing to (1), we see that even if the grass lost no water to evaporation or transpiration, it would be close to being limited in its growth by the amount of H available from H_2O. In fact grass and trees ,like humans, lose most of their water through transpiration. Desert plants like cacti have evolved to lose much less.

2. How many Mbytes of disk space would be required to store all the books and journals ever written? Does this much disk space (let's call it DS) exist, in either localised or distributed form? Large cosmological n-body simulations fill the memory of the Intel Paragon on campus. How long would it take the Paragon to generate an amount of data equal to DS, if one wrote out all the data every 100 time steps? Discuss the difference between useful and useless information.

   Library of congress has 2.5×10^7 books and journals (in all languages). Suppose that represents 1/10 of everything ever written, so about 2×10^8 books, each of 200 pages, with 60 lines/page and 60 letters/line, or about 7×10^5 letters per book. In uncompressed ascii format, each character is a byte. So a round 10^6 bytes/book. If we compressed (factor of 2.5 or so) that same 10^6 bytes/book could have room for some graphics if pictures are worth a thousand words. Real high resolution graphics, even compressed (JPEG), are more like 50kbytes, so if we wanted to store copies of all the pictures in books (not just the useful information in the picture), we might want to aim for 10^7 bytes/book (most books aren't picture books). So about 2×10^{14} bytes or 2×10^5 Gbyte without pictures, 10 times that with pictures. The largest juke-boxes I know about are about 100Tbyte= 10^5Gbyte, so they could just about hold the written material. There are probably 2000 workstations on campus, each with $\sim$ 1Gbyte disk on line. Say 50 such campuses in the US, plus the super-computer centers with their juke-boxes. So the current US research universities probably have about enough disk space to store all the books and pictures.

   Paragon has 100Gflop/s, 4Gb=1Gword memory. Suppose 100 flops/word/timestep. 10^4Gflop per 100 timesteps write, so write at 4Gb/100s=40Mb/s (just about sustainable on the disk banks). So takes 5×10^6s (2 months) to write all the books without pictures, or 1.5 years to write all the books with pictures. But would you really trade that data-set in for Faust, Fauré's music, Flaubert's novels, Frost's poems and Feynman's lectures, to name just a few of those 10^8 books?

3. How much warmer is a big city [say 10^7 people in a square 20km on a side] than the surrounding countryside? (Hint: the average American uses 10kWatt). Treat two cases:
   a) The city is trapped under a breeze-less inversion layer, so all heat must be radiated.
   b) The heat is convected up into the atmosphere and carried away by horizontal winds.

   a) The energy produced per unit area is $10^{18}\text{erg}/(2 \times 10^6\text{cm})^2 \approx 3 \times 10^5 \text{erg s}^{-1}\text{cm}^{-2}$. If this energy is radiated, this number must be equal to $\sigma T_{\text{city}}^4 - \sigma T_{\text{country}}^4$. If $T_{\text{country}} \approx 300\text{K}$, then the difference in temperature will be $T_{\text{city}} - T_{\text{country}} \approx 40\text{K}$!

   b) Assuming that the wind blows at 20km/hr, there will be a city-hour of heat in the city at all times, so we will have $\Delta E = 10^{18}\text{erg/s} \times 3600\text{s} \approx 4 \times 10^{21}\text{ergs}$. To find out the temperature difference, we note that $\Delta E = Nk\Delta T$, where N, the number of particles, is $N = \rho V/(28 \times 1.7 \times 10^{-24}\text{g})$. If the heat is mixed by convection to a height of 1km, the volume is $V \sim 400\text{km}^3 = 4 \times 10^{17}\text{cm}^3$. With a density of 10^{-3}g/cm^3, the temperature rise would be about 3K. This is roughly what is observed in urban "heat islands". Get out of New York City in summer, especially on days without breezes!

4. Light bulb filaments are made of refractory metals (e.g. Tungsten) so that when heated enough to radiate at optical wavelengths they don't sublimate.

a) The resistance of a light bulb measured with a 3 V battery tester is about 10 times lower than it is when measured at 120 V line voltage. Why? Can you think of a consequence from your personal experience?

b) Predict the length and thickness of the filament of a 100 W light bulb.

a) The resistivity is due to the scattering of electrons by phonons so it is proportional to temperature. The filament is at 300 K when a 3 volt battery is across it and at 3000 K under normal operation at 120 volts. Thus the power dissipated in the filament $P = V^2/R$ is 10 times higher when it is first turned on from a cold state. That is why bulbs almost always blow (with a blue flash: briefly reaches 6000K and sublimates like mad!) when you turn them on, not while they are on steadily.

b) Symbols: P power, V voltage, R resistance, ρ resistivity, ℓ length, r cross sectional radius, T temperature, $\sigma \approx 6 \times 10^{-12}$watts deg^{-4}cm^{-2} Stefan-Boltzmann constant.

$$P = \frac{V^2}{R} = 2\pi r \ell \sigma T^4,$$

with $R = \rho\ell/2\pi r^2$ yields

$$\frac{\ell}{r^2} = \frac{\pi V^2}{\rho P} \quad \text{and} \quad r\ell = \frac{P}{2\pi\sigma T^4}.$$

Solving for r and ℓ using the numerical values $\rho \sim 2 \times 10^{-5}$ohm cm for the *high temperature* resistivity of tungsten, and $T \approx 3000$K for the operating temperature of the filament, we obtain

$$r \sim 2 \times 10^{-3}\text{cm}, \quad \text{and} \quad \ell \sim 20\text{cm}.$$

Sterl has verified this by smashing a General Electric 100 W, 120 V lightbulb. To the naked eye, the coiled filament looked much thicker than the estimate above. However a microscope shows that what the naked eye interprets as the filament is in fact a very thin coil of wire, the wire being about 40μm in diameter. Thus the filament is a coiled coil, with 20 little coils (diameter 200μm) per big coil (diameter 0.1 cm), and 34 big coils in the filament, for a total length of $34 \times 20 \times \pi \times 2 \times 10^{-2}$ cm $= 40$ cm. Order of magnitude physics triumphs again!