# Physics 103c

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Order of Magnitude Physics

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## Problem Set 3

Due at *beginning* of class *Wednesday, 26 April 1995*

**Homework Problems:**

1. Electromagnetic communication with submarines is accomplished via low frequency (5-100 hertz) radio waves. What is the penetration depth for the radio waves below the surface of the ocean at 5Hz and at 100Hz? The resistivity of seawater is on Purcell's sheet.

2. The water in a Japanese bath is about $\Delta T \approx 6$ degrees centigrade hotter than body temperature. When submerged up to your neck in such a bath,
   a) at what rate (in watts) does heat flow into your body provided that:
      i) you move around at 1 meter per second,
      ii) you remain motionless for 5 minutes?
   b) how does your body manage to maintain its temperature at a safe level?

   Useful information: The Prandtl number for water, $Pr \equiv \nu/\kappa \approx 6$, where $\nu$ is the kinematic viscosity and $\kappa$ is the thermal diffusivity.

3. Rare earth alloy magnets have permanent magnetisations corresponding roughly to the permanent alignment of a few electron spins (magnetic moment=Bohr magneton) per atom. If two such magnets, roughly cubical in shape, are allowed to pull each other together from a large separation, what is the ratio of their kinetic energy at the moment of impact to their total atomic binding energy [hint: you should be able to express your answer just in terms of powers of the fine structure constant, and dimensionless factors of order unity]? Do you think they are likely to break when they hit?

4. Magnetic fields maintained by the motion of conducting fluids (fluid dynamos) abound in nature. Dynamo theory is a well-developed branch of applied physics. However, the corresponding experimental subject does not exist. Can you explain why? Hint: The kinematic dynamo equation reads

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \kappa_M \nabla^2 \mathbf{B}$$

   By itself, the first term describes the motion of magnetic field lines frozen into perfectly conducting fluid. Kinetic energy is transferred into magnetic energy as fluid motions stretch field lines. This is the crux of dynamo action. The second term arises from ohmic dissipation. It describes the diffusion of magnetic field lines out of imperfectly conducting material. Form a dimensionless ratio known as the magnetic Reynolds number, $R_M$, which describes the relative importance of the dynamo to the dissipative term. Estimate the largest value of this number that one might achieve in a laboratory experiment on Earth.

5. At energies far below 100 GeV, weak interaction amplitudes depend on the Fermi constant $G_F = 1.4 \times 10^{-49}$ erg cm$^3$ [historical note not needed for the problem: Fermi worked out this problem in 1935, with great success; the Weinberg-Salam-Glashow theory in 1972 related $G_F$ to the mass $M_W$ of a hypothetical $W$ particle by $G_F \simeq 4\pi(e\hbar/M_W c)^2$, also with great success.] Physical decay rates, cross-sections, etc., depend on the square of amplitudes, and hence are linear in $G_F^2$.
   a) The weak beta-decay rates of mirror nuclei (neutrons become protons and protons become neutrons) have dimensionless matrix elements of order unity. The decay rates ($1/\tau$, where $\tau$ is the half-life) depend on $G_F$, Planck's constant $\hbar$, the speed of light $c$, the energy $E$ released in the decay, and the electron mass $m_e$. Use the Buckingham Pi theorem to identify all the independent dimensionless quantities.
   b) For $E \gg m_e c^2$, the weak decay rate does not depend on $m_e$. Use this fact, the information given in the statement of the problem, and the Pi theorem determine a formula for the half-life to beta decay, and use this formula to compute the neutron's half-life for beta-decay to a proton ($E = 0.8\,\mathrm{MeV}$ —the formula works well even for $E \sim m_e c^2$).
   c) The actual neutron half-life is 10 minutes. How big is the dimensionless number missing from your formula? Would it have helped if you had used $h$ instead of $\hbar$ in your order-of-magnitude estimate?

6. Invent a problem of your own.