Physics 103b

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Order of Magnitude Physics

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Problem Set 2

Due at *beginning* of class *21 January 1997*

Homework Problems:

1. Could Bill Gates buy Pasadena?*

2. Estimate the mass of rubber liberated from car tires each year by the cars travelling along the stretch of the 210 freeway passing through Pasadena.

3. Light bulb filaments are made of refractory metals (e.g. Tungsten) so that when heated enough to radiate at optical wavelengths they don't sublimate.
   a) The resistance of a light bulb measured with a 3 V battery tester is about 10 times lower than it is when measured at 120 V line voltage. Why? Can you think of a consequence from your personal experience?
   b) Predict the length and thickness of the filament of a 100 W light bulb.

4. In class, by considering the energy of extra bonds per neighbor at the surface, we estimated the surface tension of a liquid.
   a) Using the same procedure and what you know of nuclear binding energies, estimate the surface tension of the nuclear fluid. The radius of an atomic nucleus of atomic number A is $\sim 1.2 \times 10^{-13} A^{1/3}$ cm.
   b) For heavy nuclei, considerations of nuclear physics require that the number of protons Z is given by $Z/A \sim 0.4$. Show that the electrostatic plus surface energy of nuclei with $A < A_{\rm max}$ is increased by fission (splitting the nucleus into two halves), so fission will not occur spontaneously, but that for $A > A_{\rm max}$ the energy is reduced by spontaneous fission, and estimate $A_{\rm max}$. Your answer will be improved if you estimate the electrostatic energy carefully.
   c) Estimate the maximum possible angular momentum of a tin nucleus (mass number $A = 120$, atomic number $Z = 50$), in units of $\hbar$.

* Historical note: In the not-too-distant past, Pasadena *was* owned by one person. In the years before and after 1848, when Mexico ceded California to the Union, the 14,000 acre Rancho el Rincon de San Pascual was owned by Colonel Manuel Garfias. Garfias lost title to the Rancho in 1859 to Dr. John S. Griffin and Benjamin "Don Benito" Wilson when he failed to make payments on a loan they had made him. In 1873 Wilson sold his portion to a real estate development association, which subdivided the land and named the community "Pasadena".

5. [Note: to answer this question, you don't have to know any thermodynamics, though you'll appreciate it more if you do. Enthalpy has units of energy.] The enthalpy per particle $h = H/N$ of a relativistic gas containing N particles in a volume V depends on:
   $s = S/N$, the entropy per particle,
   p, the pressure,
   $\hbar$, Planck's constant,
   c, the speed of light,
   m, the rest mass of the particles in the gas,
   k, Boltzmann's constant.
   [Why not temperature T, density $n = N/V$, etc. too? Because any two variables suffice to specify the thermodynamic state of a system. For use with h, s and p are the two natural variables, but (almost) any other pair would do]
   a) How many independent dimensionless quantities Π_i can be formed from these 7 variables?
   b) One of these is $\Pi_1 = s/k$. Find all the others, and give an expression for H in the form $H = Nv_1^{\alpha_1}v_2^{\alpha_2}\phi(\Pi_1, \Pi_2, \ldots)$, where v_1 and v_2 are two of the variables listed above. The function ϕ cannot be determined from dimensional analysis alone (and in fact also depends on the particles' dimensionless spin).
   c) If the gas is nonrelativistic, the speed of light is no longer a relevant variable, so c must cancel out of the equation in (b). Give the resulting equation for H, and using the thermodynamic relation $\partial H/\partial p = V$, use that equation to prove that for any nonrelativistic gas $H = (5/2)pV$.
   d) If the gas is ultrarelativistic, c will be relevant, but the rest mass of the particles shouldn't matter. Give the resulting equation for H, and prove (as in (c)) that $H = 4pV$ for any gas of ultrarelativistic particles.

6. Invent a problem of your own (you don't have to know the answer). The most interesting problems submitted will be done in class, or assigned as homework in subsequent problem sets. Your problem can be like those above, or more general.