Physics 103b

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Order of Magnitude Physics

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Problem Set 9

Due at *beginning* of class *11 March 1997*

1. Writing with Pencils
   a) Perform an experiment to estimate the thickness, in atoms, of the graphite layer left by a pencil writing on a piece of paper.
   b) Provide a theoretical estimate based on the material presented in class.

2. Consider a mass $m = 1$kg hung from a massless string of length $\ell = 1$m (a simple pendulum). Estimate the rms angular displacement of the string from vertical due to:
   a) quantum fluctuations,
   b) thermal fluctuations at $T = 300$K.

3. Free-Free Absorption
   a) Derive an approximate analytic formula giving the absorption length for an electromagnetic wave of radian frequency ω propagating in a completely ionized hydrogen plasma of number density n and temperature T. Proceed by calculating the energy stored in the oscillations of the electrons forced by the incident radiation. Assume that this energy is dissipated as heat during strong electron-proton collisions during which the electron is deflected by an angle of order a radian or greater.
   b) A massive star is surrounded by a sphere of ionized hydrogen of of radius $R = 10^{18}\,\mathrm{cm}$ with $n = 10^3\,\mathrm{cm}^{-3}$, $T = 10^4\,\mathrm{K}$. What is the critical wavelength λ above which the sphere is opaque to electromagnetic waves?

4. Piano strings
   a) The highest note on a grand piano (C8, 4186Hz) is a steel string of 5 cm length. Estimate the strain ϵ of this string. Are you impressed by the quality of steel required? Increasing the string tension increases the forces exerted on the sound board, and thus the maximum loudness of the piano.
   b) What is the Mach number (in air) of the transverse waves on the piano wire of (a)?
   c) The string of the lowest note (A1, 27.5Hz) on the grand piano is 2m in length; it consists of a steel core under tension, surrounded by a copper winding (under no tension) with diameter double that of the steel core. Estimate the strain ϵ of this string. Only the strings of the lowest 20 notes on the piano are wound in this way. Can you see why?
   d) The upper strings are coupled to the lower ones through the pins and soundboard, so when a low string is struck, all upper strings resonant with its harmonics vibrate

sympathetically. To minimize beating between these sympathetic vibrations of the upper strings (or their primary vibrations if they are played as part of a chord!) and the overtones of the lower strings (recall the inharmonicity due to stiffness), the notes below middle C are tuned progressively flatter, and notes above tuned progressively sharper (Railsback curve). In a properly tuned piano, C8 is tuned a factor 1.017 higher than the 16th harmonic of C4 (middle C). Estimate the diameter to length ratio a/L and the string tension (in kg or pounds force) of the middle C string, which is about 80cm long.

5. Generation of Sound by Turbulence

   a) Consider three-dimensional fluid turbulence with characteristic velocity v, outer scale L (the scale on which the turbulent motions are driven), and Mach number $M \sim v/c_s \ll 1$. What is the approximate amplitude of the turbulent pressure fluctuations?

   b) Estimate the efficiency of acoustic radiation by the turbulence. Express the power radiated per unit volume as a fraction of the total energy dissipation rate per unit volume, $\mathcal{E} \sim \rho v^3/L$.

      Hint: Quadrupoles are the lowest order acoustic multipoles for free turbulence (can you see why?).

6. Make up a problem of your own.