Ph 103b Final Exam

DUE before 2:00pm Wednesday March 19, 1997

Your completed exam should be handed *in person* (do *not* stick the exam under doors) to one of the following people: Shirley Hampton (151 Bridge Annex), Kay Campbell (150 South Mudd).

During this exam, you may not refer to anything other than these sheets of paper and your brain. In particular, you may *not* consult books, notes, homework sets, or any other papers, nor may you consult computers or human beings. You may use a non-programmed calculator. A copy of the Purcell handout of numerical values is inside this exam.

You may work on this exam for a continuous interval of up to 6.0 hours. The exam has 90 points total. There are six short questions (5-8 points, as marked) and five long questions (10-12 points each). The problems are ordered in order of difficulty. You are also requested to make up a problem of your own (5 points). For this, you may either re-use the best of the problems you made up for the homeworks, or make up an even better one (you may think about this before the exam!).

Please write the exam in a Blue Book (available in the bookstore).

Physics 103b Final Exam

P. Goldreich & E. S. Phinney

Shorter Problems

1. [5 points] To lift your body out of the swimming pool, you must overcome the forces of both surface tension and gravity.
   a) Estimate the ratio of these two forces.
   b) Repeat the calculation for a fly in the pool.

2. [5 points] How does the incident acoustic energy flux of a whisper compare to the incident visible electromagnetic energy flux from a 5'th magnitude (faint naked eye) star? Useful facts: i) a whisper is about 20 decibels where the sound level in decibels is related to the flux of acoustic energy $\mathcal{P}$ by $10\log(\mathcal{P}/10^{-12}\mathrm{watt\,m^{-2}})$; ii) the sun would appear as a 5'th magnitude star if it were at a distance of $10\,\mathrm{pc} \approx 3\times 10^{19}$ cm.

3. [5 points] The electron is the lightest member of the family of 3 leptons which also includes the muon and tauon. Suppose the world were made up of muons and nucleons instead of electrons and nucleons. Describe one aspect in which it would differ from the world as we know it. Note: $m_\mu \approx 207 m_e$.

4. [5 points] Estimate the thickness a one meter by one meter glass window pane must have in order to withstand winds of $100\,\mathrm{km\ h^{-1}}$.

5. [5 points] Consider the stability of a two dimensional, incompressible, stratified, shear flow. Let z denote the vertical coordinate and g the constant gravitational acceleration. The flow is characterized locally (in z) by the logarithmic density gradient, $d\ln\rho/dz < 0$, and the vertical shear of the horizontal (z) velocity, du_x/dz.
   a) Provide a dimensionless parameter whose value determines the stability of the flow. Do not introduce any variables other than the ones listed above; the problem is purely local, so z is irrelevant.
   b) What is the physical significance of this parameter?

6. [8 points] A particle of speed $v = \beta c$ moving through a medium with index of refraction n emits Cerenkov radiation if $\beta > 1/n$. The energy radiated in Cerenkov radiation at frequencies between ω and $\omega + d\omega$, per unit length traversed by the particle, $dE(\omega)/dx$ depends on the electron charge e, speed of light c, ω, n, β, and is linearly proportional to $d\omega$.
   a) Use the Buckingham Pi theorem to find a general functional expression for $dE(\omega)/dx$.
   b) Recall that at frequency ω, the index of refraction for a medium composed of atoms whose electrons are approximated as harmonic oscillators of natural frequency ω_0 is given by $n^2 = 1 + 4\pi N e^2/(m_e(\omega_0^2 - \omega^2))$, where N is the number of electrons per unit volume. For ultrarelativistic particles, the $dE(\omega)/dx$ of Cerenkov radiation is independent of β, and in a dilute medium such as air, with $n-1 \ll 1$, $dE(\omega)/dx \propto N$. Find an expression for $dE(\omega)/dx$ in this case ($\beta \to 1$, $n-1 \ll 1$).

7. [5 points] Invent a problem of your own.

Long Problems (10 points each)

1. [10 points] Fun with turkeys

   A mad scientist stuffs his Thanksgiving turkey with liquid oxygen, attaches a nozzle and ignites it; the resulting burn lasts several seconds. Estimate the maximum altitude the charred remains could attain

   a) if the earth had no atmosphere.

   b) including the effects of the earth's actual atmosphere.

   c) in case (b), how does the altitude attained scale with the size of the turkey?

2. [10 points] Toaster physics

   An american (120V) toaster uses 400W in each bread slot. Each slot is wound with 300cm of metal ribbon 10^{-2} cm thick, which serves as the heating element.

   a) Estimate the width of the metal ribbon.

   b) Estimate (using fundamental physics and the numbers provided) the temperature of the ribbon when the toaster is on.

   c) How long does the ribbon take to reach this temperature after the toaster is turned on?

3. [12 points] The blood and guts of exercise

   You are pedalling your bicycle as hard as you can, generating 300W of mechanical power.

   a) If there were no heat flow out of your leg muscles, how long would it take for the temperature deep inside your leg to rise by 1 K (onset of fever)?

   b) If the inner temperature of the leg did rise by 1 K, what fraction of the heat generated by the leg during vigorous exercise could be carried to the skin by static thermal conduction in the leg tissue?

   c) The excess heat is carried from the interior to the skin by the blood vessels. Estimate the volume of blood flow (in $\mathrm{cm^3\,s^{-1}}$) required to transport the heat generated by the legs during exercise.

   d) Estimate the mass of oxygen per unit time ($\mathrm{g\,s^{-1}}$) which must be carried to the muscles to sustain the exercise aerobically.

   e) 50% of your blood volume is occupied by red blood cells. 1 $\mathrm{cm^3}$ of packed red blood cells contain 0.35 g of hemoglobin. Oxygenated hemoglobin contains 1 O_2 molecule. Use this information, combined with your answers to parts (c) and (d), to estimate the molecular weight of hemoglobin.

   f) Your white (anaerobic) muscles store enough energy to sustain maximum output for about 30 seconds. Would there be a point to storing more energy?

4. [10 points] Interplanetary Sailing Using Solar Radiation

   A circular sail composed of aluminum foil is attached to a spacecraft of equal mass in a manner similar to a parachute (i.e. the spacecraft hangs in the center of the sail on the sunward side, suspended by threads which extend from the circumference of the sail). Assume launch from earth orbit (1AU $= 1.5 \times 10^{13}$ cm from the sun). The ram pressure of the solar plasma wind is much less than the solar radiation pressure.

a) What is the maximum thickness of the aluminum foil which would permit sailing in the radial direction away from the Sun?

b) What is the maximum radius that the solar sail composed of a single sheet of aluminum could have if it is to avoid tearing? Consider the system with the thickness calculated in part (a).

c) Estimate the minimum thickness of aluminum foil sufficient to reflect most solar radiation.

5. [10 points] Bombs and torpedoes

An explosion in an isotropic medium (like air or water) creates a spherically expanding shock front, across which the pressure increases discontinuously. A sudden overpressure of 0.1–1 atm (equivalent to 0.1–1 kg weight per cm^2) destroys living things, buildings and ships.

a) While the velocity of the shock is much larger than the speed of sound c_s of the medium through which it propagates, the shock speed v depends only on the density ρ of the medium, the instantaneous radius R of the shock, and the explosion energy E. Use the Buckingham Pi theorem to find the shock speed v as a function of E, R and ρ.

b) When the energy density (pressure) behind the shock drops below ρc_s^2, it ceases to be a strong shock, and its speed is no longer given by the v you found in part (a), but it instead propagates at nearly the sound speed as a (sharp-edged) wave. Find the postshock pressure as a function of E, R, ρ and c_s in this limit.

c) Consider two explosions of identical energy E, one of which occurs under water and the other in air. Compute the ratio of the distances at which the shockwave pressure falls to 1 atm, R_{1w}/R_{1a}, and thus show that underwater explosions are destructive to a much larger distance than the same explosions in air.

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