Textbook proposal

Order-of-Magnitude Physics: Understanding the World with Dimensional Analysis, Educated Guesswork, and White Lies

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Proposal contents

Summary. Even after years of our teaching, physics students do not think like physicists. They cannot apply their knowledge to unfamiliar problems; they cannot understand the world around them. Two of us have taught a course in order-of-magnitude physics at the California Institute of Technology for the past 11 years. This book, which has grown out of the course, teaches the art of approximation: dimensional analysis, guessing, and lying. To illustrate these techniques, we study the physics of everyday phenomena—rainfall, flight, radio waves, sleeping bags, and much else. The audience for the book includes graduate students and upper-level undergraduates in the physical and mathematical sciences, as well as practicing engineers and physicists. We present a resource of reasoning methods to transform diligent readers into physicists.

Why do AM radio waves travel farther at night than during the day? What is a typical annual rainfall? Why does a tea kettle whistle, and how loud is the sound? How long must we leave a freezer door open to defrost a 3-inch layer of ice?

1 Why bother?

Physics students—after a decade of training in high school, university, and graduate school—usually cannot identify the physical principles required to answer these questions. They cannot think like physicists. The students lack not facts but rather a mastery of principles. This difficulty forces students into parrot learning; physics becomes for them a game of formula juggling. To avoid this problem, we should teach students how physicists think: how we approximate and reason in unfamiliar situations.

With that goal in mind, Peter Goldreich and Sterl Phinney have taught a course in order-of-magnitude physics at the California Institute of Technology (Caltech) for the past 11 years. The proposed book is based on lecture notes from the course. Sanjoy Mahajan was twice a teaching assistant for the course, and has primary responsibility for writing this book. We expect a complete draft by January, 2000.

The *Order-of-Magnitude Physics* textbook applies general physical principles, scaling relations, and dimensional arguments to the physics in our surroundings: weather, materials, sound, exercise, light, earthquakes, and more. It illustrates these physical principles with examples, and it teaches a rough-and-ready style of reasoning. It transforms diligent readers into physicists.

2 An example

As an example of order-of-magnitude physics, let's estimate a typical annual rainfall (this example is discussed in Chapter 7). The analysis uses simple principles, but demands a readiness to approximate that is unfamiliar to most students.

Sunlight evaporates water from the oceans; the water rises up and falls back as rain. One square centimeter of the Earth receives approximately 0.1 W, or 0.1 J/s, of solar energy. In Chapter 4, using a few physical principles and a few multiplications, we estimate the energy required to vaporize water: 2500 J/g. If sunlight evaporates water with perfect efficiency, then it would take roughly $3\times10^4\,\mathrm{s}$ to evaporate 1 g; this mass corresponds to a volume of $1\,\mathrm{cm}^3$ or to a thickness of 1 cm over the $1\,\mathrm{cm}^2$ area. A year is $3\times10^7\,\mathrm{s}$ (conveniently and accurately remembered as $\pi\times10^7\,\mathrm{s}$). In that time, the thickness that evaporates is

$$\frac{3\times10^7\,\mathrm{s}}{3\times10^4\,\mathrm{s/cm}} = 1000\,\mathrm{cm}. \qquad (1)$$

This estimate is a factor of 10 too large, but because the derivation is so simple, correcting it is simple. First, sunlight is only roughly 30 percent efficient in evaporating water: Much of the energy heats up air and land without evaporating water. This percentage, while hard to estimate from first principles, is a typical efficiency; internal-combustion engines and mitochondria are roughly 25 percent efficient in turning fuel into mechanical work. Second, the solar flux of $0.1\,\mathrm{W/cm}^2$ applies only at the equator, and even there only during the day. At higher latitudes, particularly during the winter, sunlight strikes at a slant, which reduces the incident energy. Averaging the flux over the whole surface, including correcting for the existence of night, we should divide the solar flux by 4. The two corrections reduce our rainfall estimate by a factor of 10: In the order-of-magnitude world, $4/0.3 = 10$. The annual rainfall is then 100 cm/year, which is reasonably accurate. This estimate applies to a world covered purely with ocean; the rainfall over land, which depends on details such as mountains, coastline, and forest, varies enormously.

After a few years of studying physics, students know the two principles required to estimate rainfall: heat of vaporization and solar flux. These principles, and some primary-school arithmetic (division), determine the rainfall; but only the most insightful students make the approximations needed to solve the problem. Our book teaches these techniques and makes all students into insightful students.

3 Origins

The book is based on a one-quarter (26 lecture-hours) course given in alternate years at Caltech since 1986. The students are third- and fourth-year undergraduates, graduate students, and postdoctoral researchers. Sanjoy Mahajan started with the lecture notes that have been class tested over 11 years, adding examples and extra explanation for the points that have confused students. Lyn Dupré copyedited many of the chapters.

The text consists of many short derivations that use no calculus. After the derivations, we include numerical values from the real world, to develop students' feel for magnitudes, and we include applications to everyday physics. For example, after estimating the thermal conductivity of insulators (Section 4.6.1 in the sample chapters), we check it against a table of the thermal conductivities of glass, rock, and wood.

The structure of the derivations teaches order-of-magnitude techniques. Labeled braces mark important terms or groups of terms in the derivations, as in the annual-rainfall example:

$$\text{Rainfall} = \frac{\overbrace{0.1\,\text{W/cm}^2}^{\textit{solar flux}}}{\underbrace{3000\,\text{J/g}}_{\substack{\textit{heat of}\\ \textit{vaporization}}} \times \underbrace{1\,\text{g/cm}^3}_{\textit{density}}} \times \underbrace{3\times10^7\,\text{s/year}}_{\substack{\textit{convert to}\\ \textit{annual rainfall}}} = 1000\,\text{cm/year}. \tag{2}$$

The braces and their labels teach students how to read an equation, not as a sequence of characters but as a group of ideas. Equation reading, once it becomes a habit, prepares students for a creative task, equation writing, essential for understanding unfamiliar phenomena. These skills of reading and writing are developed in the text through many examples.

The students in the class solve five problems per week—problems drawn from a collection that we have accumulated over years of teaching. These and other problems are sorted by chapter, and are divided into *Warmup*, *Homework*, *Exam*, and *Research* problems; this kind of subdivision worked well in *Concrete Mathematics* by Graham, Knuth, and Patashnik. The warmup problems simply help the reader to pay attention to the ideas. The homework and exam problems extend the ideas presented in the chapter, with the exam questions requiring the longer chains of reasoning. The research questions are unsolved; we welcome any solutions. We are considering whether to include problem solutions or only hints. The text contains many worked-through examples, which serve as model solutions. The sample chapters do not yet contain problems, but we have included the problem and solution sets from the 1995 course. Much of our course material, including the 1997 problem and solution sets, is available on the Internet at `http://hope.caltech.edu/oom`.

4 Audience

The audience for the book includes the same groups of students who took the course: graduate students and upper-level undergraduates in the physical and mathematical sciences. It also includes readers of *The Flying Circus of Physics* by Jearl Walker (New York: Wiley, 1977), because of our shared emphasis on everyday physics.

The audience extends beyond the university to include practicing physicists and engineers. Low-temperature physicists estimate heat flows (Chapters 4), for example to calculate helium consumption. Mechanical engineers' daily bread is struts and beams, which are discussed in Chapter 10; biomedical engineers will enjoy the applications of physics and engineering to biology. Electrical engineers will appreciate the treatment of radiation (Chapter 6). And all engineers use dimensional analysis (Chapter 2 and throughout the book).

Order-of-magnitude physics is taught at Caltech and at institutions seeded with Goldreich and Phinney's former graduate students, who take handwritten notes with them. The students in these courses would benefit from our book. Harvard also has a similar course, *Widely Applied Physics*, which concentrates more on astrophysics and less on everyday physics. Physics graduate students studying for their qualifying exams, an audience of roughly 2000 every year in the United States alone, will find our book useful preparation. Most qualifying questions require a confidence with approximations and physical reasoning, and examiners test those skills using problems similar to ours.

Physics faculty around the world complain that students may have learned but cannot use physics. Therefore the physics courses in Cambridge, Edinburgh, and many other UK universities include a general-physics examination, with questions drawn from anywhere in the syllabus (some US universities and four-year colleges use a similar examination). Students have great difficulty preparing for the exam, and departments have great difficulty helping them. The order-of-magnitude book will help both students and their teachers.

We believe that there is great demand for an order-of-magnitude physics course, but the demand has not been met because there is no book. At Caltech, 60 students take the course each time—a large class at a small institution such as Caltech, which has only 1000 graduate students. Many graduate students from the course mention how they wish their undergraduate university had taught such a course. When students find out that we are writing this book, many offer to be proofreaders, and some of the departing graduate students say they will use the text to teach a class. To create such a course from scratch is difficult; the course combines many disciplines, ranging beyond physics to include examples from biology, sports, and materials science.

Our course has improved from a decade of experience. We have learned which topics grab students' interest, have invented many problems to illustrate order-of-magnitude methods, and have clarified our own understanding of the subject. The textbook will give other faculty the benefit of our experience, and make it easier for them to start an order-of-magnitude physics class. With luck, they will enjoy learning the material as much as we have. Students at institutions without such a course will use our book to help them understand the rest of their coursework. As one student in our course said, "I'm taking applied classical physics because I must, and Order-of-Magnitude so that I understand what I'm doing in that course."

5 Related books

Some excellent books that contain or emphasize order-of-magnitude methods include:

- *Gases, Liquids and Solids and Other States of Matter* by D. N. Tabor (Cambridge, England: Cambridge University Press, 1991). This book is filled with order-of-magnitude analyses of material properties, and it teaches an intuitive style of physical reasoning. It treats materials in more detail than we possibly can (we cover materials in Chapters 3, 4, and 10). This book does not cover our other topics, such as radiation, weather, and biomechanics, nor does it teach dimensional analysis and guessing. It contains no problems.

- *Understanding the Properties of Matter* by Michael de Podesta (Washington, DC: Taylor and Francis, 1996). This text is filled with scaling relations and order-of-magnitude analyses of materials, supplemented with detailed tables to compare the estimates with reality. Podesta makes a tradeoff similar to Tabor's: The text is all about materials.
- *On Size and Life* by Thomas A. McMahon and John Tyler Bonner (New York: W. H. Freeman, 1995). This beautiful book applies order-of-magnitude physics to biology. We cover similar material in Chapter 10, but in less depth.
- *Dimensional Analysis* by Percy Bridgman (New Haven: Yale University Press, 1931). This classic contains wonderful examples of dimensional analysis, and it discusses the Buckingham Pi theorem and the caveats about its use in far more detail than we do. We devote one chapter (Chapter 2) solely to dimensional analysis, and illustrate its use in examples throughout the text. We also show how dimensional analysis is part of a larger toolbox of techniques.

  For example, in Section 2.3.1 (in the attached chapters), we use dimensional analysis to determine the drag force felt by a sphere moving in fluid:

$$F = \rho v^2 R^2 f\left(\frac{vR}{\nu}\right), \tag{3}$$

  where ρ is the density of the fluid, ν is its kinematic viscosity, R is the radius of the sphere, v is its velocity, and f is a dimensionless function of its dimensionless argument. Dimensional analysis cannot determine f.

  Physical reasoning helps us to make progress, at least for low speeds (this analysis is the subject of Section 2.3.2, which expands the following argument). At low speeds, the drag is due to viscous forces, which are proportional to the viscosity, ν. Therefore F is proportional to ν as well. So f has revealed itself:

$$f\left(\frac{vR}{\nu}\right) = \text{some constant} \times \frac{\nu}{vR}. \tag{4}$$

  With this constraint, the drag force (3) becomes

$$F \sim \rho v^2 R^2 \frac{\nu}{vR} = \rho v \nu R. \tag{5}$$

  This drag force, except for the missing constant of 6π, is the expression first determined by Stokes.

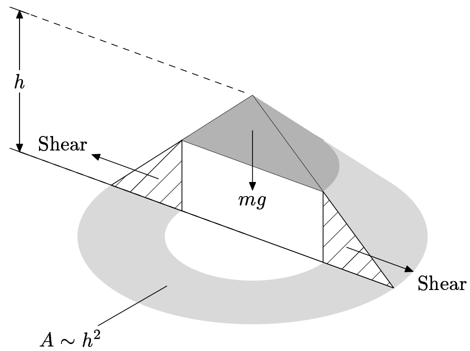
  Our mixture of techniques makes our book complementary to those purely on dimensional analysis.

For the latest Caltech course, we used *Gases, Liquids and Solids* and *On Size and Life* as supplementary texts. We would also have used *Understanding the Properties of Materials*, but we discovered it only after the course was over. Together these three books cover perhaps one-half of our book's topics.

By discussing a variety of topics, we unify the material from many physics courses—electromagnetism, quantum mechanics, statistical mechanics, and fluid mechanics—and present a coherent view of physics. For example, many physical phenomena, such as rainbows and quantum-mechanical perturbation theory are disguised versions of spring physics. Because our techniques sidestep mathematical complexity, we have time to discuss advanced topics, and can use them to illustrate a common theme.

6 Production and length

We produce the book using plain TEX (not LATEX) and draw the figures using MetaPost, a version of Donald Knuth's MetaFont program; MetaPost produces Encapsulated PostScript. The figures are line drawings and graphs; some figures use halftone shading. As a sample, here is an order-of-magnitude mountain (Figure 3.8 in the sample chapters):



MetaPost makes it easy to maintain a figure style consistent across a book. For example, if we and the book designers decide that all lines should be 0.8 points thick, instead of the default 0.5 points, we need to change only one line of MetaPost code.

The sample chapters illustrate our preliminary book design. We have borrowed elements from Edward Tufte's beautiful book *Envisioning Information*.

We estimated the full manuscript length with order-of-magnitude methods. The first five chapters, which are included with this proposal, are roughly 100 pages when set 11/14 with 2.3-inch (5.8 cm) margins for the figures and footnotes. We plan 10 chapters, so the first estimate for the book length is 200 pages. A slightly larger setting, such as 12/15, would increase the length by 20 percent (10 percent vertically and horizontally), making the book roughly 250 pages. Random pieces—such as problems, a preface, an index, and the trim size—may contribute another 50 percent. So we estimate a length around 400 pages. We shall provide film.

7 Authors

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8 Contact information

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9 Chapter summaries

1. *Wetting the feet.* We warm up with everyday estimates, make scaling analyses, and practice order-of-magnitude arithmetic. The examples introduce techniques of estimation used throughout the text (and, we hope, later in life). They include divide-and-conquer, guess, lie, cross-check, and, our favorite, lower your standards. Any quantity with dimensions cannot be inherently large or small, so we must compare it to another quantity with the same dimensions; this principle shows up in many examples.
2. *Dimensional analysis.* We introduce the Buckingham Pi theorem, dimensionless groups, and dimensional analysis. We analyze pendulum motion and compare the differential-equation solution with the dimensional-analysis solution. Then we study fluid mechanics, where dimensional analysis allows us to avoid the difficult Navier–Stokes equations. This example introduces a famous dimensionless number, the Reynolds' number, which we explain as the ratio of two speeds (always compare!); and it introduces the idea of dimensionless numbers.
3. *Materials I.* We study the mechanical properties of materials. Quantum mechanics becomes simply adding one variable, $\hbar$, to the list used in a dimensional analysis. How big are atoms and molecules? What are the binding energies of atoms and molecules? We apply our techniques to these questions, and use the answers to understand the density of solids, the speed of sound, the height of mountains, and the properties of white dwarfs. These example show how order-of-magnitude analyses span the range of physics, from the quantum to the everyday to the astrophysical.
4. *Materials II.* We study the thermal properties of materials. By making simple models, we estimate viscosities, thermal conductivities, and boiling points. The estimates provide intuitions that explain these properties. The discussion of boiling points is a poor man's introduction to the concepts of thermodynamics and statistical mechanics, in particular to entropy.
5. *Water waves.* We derive dispersion relations for water waves. The water can be shallow or deep (relative to what?); the waves can be driven by surface tension or by gravity. So we study four limits, understand the behavior in each limit, and combine the analyses to understand the general case. This chapter is an extended order-of-magnitude analysis, a large example of divide-and-conquer reasoning. Waves are an excuse to introduce a great idea of physics: Everything is a spring.
6. *Radiation.* We study the common features of acoustic, electromagnetic, and gravitational waves. These features include: a reservoir of potential energy (in electromagnetism, the electric field), a reservoir of kinetic energy (in electromagnetism, the magnetic field), and a coupling between the reservoirs. These examples of waves show again how everything is a spring, and how fortunate we are in that result—almost all of our estimates are exact, because the missing dimensionless constants in spring physics are usually unity.
7. *Weather.* Using simple physical models—flow and blackbody radiation—we estimate the annual rainfall on the earth, the thickness of clouds, and the speeds at which winds blow and rivers flow. We derive, in an intuitive way, two basic results in the theory of turbulent flows: the von Karman law of the wall (for river flow) and the Kolmogorov scaling (for turbulence in the atmosphere).
8. *Random walks and boundary layers.* Many of our estimates, such as of viscosity and diffusivity, depend on understanding random walks. We study the common features of random walks and understand how their properties depend on the number of dimensions: why you find your key ring in two dimensions but might not in three dimensions. We approximate to understand random walks, and use our results to study boundary layers: to analyze the

physics of quilts, of wind-chill factors, and of Korean restaurants where you cook thin strips of meat at the table.

9. *Brain and information theory.* We estimate the computing power of the brain and compare it to a relevant quantity, the computing power of modern microprocessors and supercomputers. How fast do nerve signals travel? How many neurons are in the brain? Did the latest chess machine Deep Blue have more computing power than Kasparov? When will computers have as much computing power as our visual cortex? How much information can we receive from the Voyager space probe?
10. *Biomechanics.* We apply physics to biology. How fast can people swim? How far can birds fly without eating? Why are there no large hummingbirds? The last two questions introduce the physics of flight, a topic shrouded in superfluous mystery. Using a scaling analysis, we compare the performance of birds and jumbo jets. We estimate the properties of bones (struts) and find out why elephants are the largest land animals.