Problem set 2

Here is the set of problems to work on for your second supervision (note that this sheet is two-sided!). As usual, discuss with each other. Staple your work. Wednesday pairs please hand in to my pigeonhole by Wednesday 10am, Friday pairs by Thursday 1pm. Email me if you have any questions. Think about the comments that I made on your work from last week, and bring it back to the upcoming supervision.

1 *Estimations*

a) How many English words can you recognise?

b) How many pieces of mail does the UK postal system carry each day? Estimate the annual budget of the Royal Mail; check your estimate by looking up a recent Royal Mail budget.

2 *Interpreting equations*

Here you will study the well-known formula for the horizontal range of a rock. You throw a rock with velocity v at an angle θ with respect to the ground. Its range is

$$R = \frac{2v^2}{g} \sin\theta \cos\theta. \tag{1}$$

You can increase your confidence in this result in a number of ways (parts a–e). (It may help for many of the parts to draw a diagram.)

a) Dimensional analysis: Check whether the dimensions are correct.

b) Consider limiting cases (for example, $\theta = 0$). Does the range formula work in these cases?

c) Give a physical argument for why the range contains a factor of v^2 (instead of, say, simply v or $1/v$ or no v at all). (Dimensional analysis, which you did in part a, is a mathematical argument; in this part, you are asked to reinforce the mathematics with a physical argument.)

d) Give a physical argument for the factor of 2.

e) Give a physical argument for the $1/g$ factor.

f) To derive (1), you have to neglect many effects (for example air resistance). List as many of these effects as you can. Let your imagination run; no effect is too small to mention here.

3 *Number sense*

Without a calculator, estimate

a) $\sqrt{1.3}$

b) $\sqrt[3]{1.6}$

c) $\sin 7$

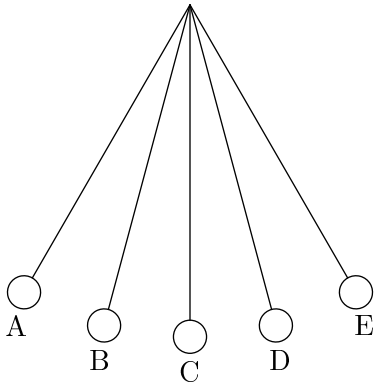
d) 1.01^{100} (Hint: What is $\ln 1.01$?)

4 *Scales*

You stand on a scale holding a book. You then place the book next to you on the scale. The two scale readings are of course identical. Of course!? Prove it by using Newton's laws and drawing free-body diagrams. Clearly label the third-law pairs (pairs that must be equal and opposite as a consequence of Newton's third law),* and describe each force in words.

5 *Pendulum*

The figure shows a pendulum at five points in its swing; positions A and E are the extremes of the motion. On each bob, draw (approximately) the acceleration vector at that point in the swing. If the acceleration is zero (in which case there is no arrow to draw), simply circle the bob.



6 *Tetrahedron*

In methane, the four hydrogen atoms lie at the corners of a regular tetrahedron, and the carbon atoms lies at the centre. Where is the centre of a tetrahedron with unit edge length? What is the bond angle (the angle between two C–H bonds)? (Hint: Make an analogy.)

* I avoid using the perhaps more familiar term 'action–reaction pairs' because it needlessly confuses; it implies, wrongly, that one force causes the other.