Problem set 5

Wednesday pairs please hand in to my pigeonhole by Wednesday 10am, Friday pairs by Thursday 1pm. Clearly explain your reasoning.

1 *Estimation: Oblateness of the earth*

Compared to a sphere, the earth is squashed.

a) Why? Should the polar radius or the equatorial radius be the larger?

b) Which physical quantities determine d, the difference in radii? How can you combine these quantities into a length (in other words, into an estimate for d)?

c) Use your formula to make a rough numerical estimate of d, and compare it with actual data.

2 *Moments of inertia*

a) What are the dimensions of moment of inertia?

b) An object has mass M and characteristic length l. The characteristic length is a typical length in the object, such as a radius or diamater. What is its moment of inertia, up to a dimensionless constant? Consider a geometrically similar object that is twice as big as this object, in all its dimensions, and made out of the same material. What is the ratio of moments of inertia: $I_{\text{bigger}}/I_{\text{smaller}}$?

c) The moment of inertia of a uniform thin disc is $MR^2/2$, about an axis perpendicular to the plane of the disc and through its centre. Perhaps using your results from last week, guess a moment of inertia for a uniform spherical shell with mass M and radius R (axis of rotation through the centre). Now calculate it and compare with your guess.

3 *Rolling*

Four objects, made of identical steel, roll down an inclined plane. The objects are (1) a large spherical shell, (2) a large disc, (3) a small solid sphere, and (4) a small ring. The large objects have triple the radius of the small objects. Rank the objects in order of decreasing acceleration down the plane.

4 *Buoyancy*

A solid iron sphere is floating in a bath of mercury. You pour water over the sphere and cover it with water. Does the sphere rise, sink, or stay at the same height?

5 *Quadratics by approximation*

a) Use the quadratic formula and your calculator to find the solutions of $1 + 200000x + x^2 = 0$. What goes wrong? Why?

b) Instead, let's approximate. If x is near zero, which term can you neglect? Solve the simplified equation to get a first approximation to the smaller root. Call this first approximation x_1.

c) How can you improve your approximation?

d) If you know one root, how can you easily find the other root?