

1 *Weighings*

You are given 12 balls, all of which are equal in weight except for one which is either heavier or lighter. You are also given a two-pan balance to use. In each use of the balance you may put any number of the 12 balls on the left pan, and the same number on the right pan, and push a button to initiate the weighing; there are three possible outcomes: either the weights are equal, or the balls on the left are heavier, or the balls on the left are lighter. Your task is to design a strategy to determine which is the odd ball *and* whether it is heavier or lighter than the others *in as few uses of the balance as possible*.

2 *Walking speed*

What limits how fast you can walk? How fast can a horse walk?

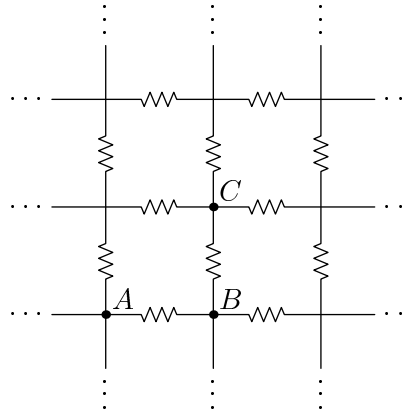
3 *Energy puzzle*

A ball bounces elastically off a wall (in one dimension): It strikes and rebounds with velocity v_0 , and energy is conserved. In a reference frame that moves with the initial velocity of the ball, the ball before the bounce has velocity 0; and after the bounce, velocity $2v_0$. Energy is not conserved.

4 *Resistive grid*

a) In the (infinite) two-dimensional lattice of $1\ \Omega$ resistors shown below, what is the resistance between points A and B ?

b) (Hard!) What is the resistance between points A and C ?



5 *Ice pole*

A pole sits vertically in a sheet of ice (say, on a pond), and a string is attached to the pole and laid outwards (see figure below). A hockey puck slides along the ice (no friction of course) and hooks onto the the string. Initially the puck is moving with velocity v (all tangential). When the rope is half wound round the pole (so the rope is half its original length), how fast is the puck moving? There are at least two plausible answers, but at most one is right!



6 *Tricky die*

Lacking any vacation problems to work on, you roll a 1000-sided die once per second.

- a) How long, on average, between rolls of a 1? *Answer:* 1000 s
- b) Your friend Jane walks up and sees you rolling the die. How long does she have to wait, on average, before a 1 turns up? *Answer:* : 1000 s (careful of the gambler's fallacy)
- c) How long, on average, between the time that she walked up to you and the time that you *last* rolled a 1? *Answer:* : 1000 s

Combining the answers to parts b and c, we conclude that a 1 turns up every 2000 s, in contradiction to part a. You decide!

7 *Trees*

Atmospheric pressure supports a column of mercury 0.76 m high, and a column of water 9 m high (because water is 11 times less dense than mercury). How do you explain trees taller than 9 m?

8 *Fever*

You have just tested positive for BSE, which kills 100 people per year in the UK. In this hypothetical test, anyone who has BSE tests positive with probability 1, and anyone who does not tests positive with probability 0.01. How concerned should you be? (What is the probability that you have BSE?)

9 *Milk bottle*

You have a (now mostly historical) full glass milk bottle with the cream floating on top. The bottle is on a weighing scale. You pick it up and shake it thoroughly, mixing the cream and milk. Does the pressure at the bottom of the bottle increase, decrease, or remain the same? Does the weight increase, decrease, or remain the same?