1 How to value heat

Introduction and overview

Author: David MacKay,
Chief Scientific Advisor, DECC
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Intended audience: people interested in heat policy, fuel policy, and combined heat and power.

Which has more value –

(a) 1 cup of boiling water and 9 cups of ice-cold water;

or (b) 10 cups of water at 10 °C?

They both contain exactly the same amount of heat! So you can easily obtain the latter from the former by simply mixing the boiling water with the ice-water. But you can't easily turn 10 °C water into boiling water and ice-cold water.

There's all sorts of useful things you can do with boiling water and ice-cold water that you can't do with ambient-temperature water. So surely (a) is more valuable than (b).

This simple observation shows that "amount of heat" is not the right way to measure value. *How valuable heat is depends on what temperature it is at*.

Here's another thought experiment. Imagine you are in a building at 10 °C, and you have access to a tap delivering free water at 10 °C, and empty cups, if you want them. Which has more value –

(c) 1 cup of boiling water (100 °C);

or (d) or 3 cups of water at 40 °C?

Both contain identical amounts of heat. (One cup raised by 90 °C involves the same heat as three cups raised by 30 °C above the ambient temperature.) Both could be delivered using the same amount of kettle-time. Again, you can easily make the latter from the former by mixing the boiling water with two extra cups of tap-water. But you can't unmix tepid water back into cold water and boiling water! So surely (c) is more valuable than (d). This shows that *the higher the temperature some heat is at, the more valuable it is*.

This document explains these ideas and gives graphs showing the value of heat at different temperatures. The most important message is this: *no matter how high the temperature, one unit of heat is never more valuable than one unit of electricity*.

Motivation: conventional value judgements about heat

There is a widespread view that "it is better to make heat than electricity from a fuel such as biomass." This view is expressed as follows.

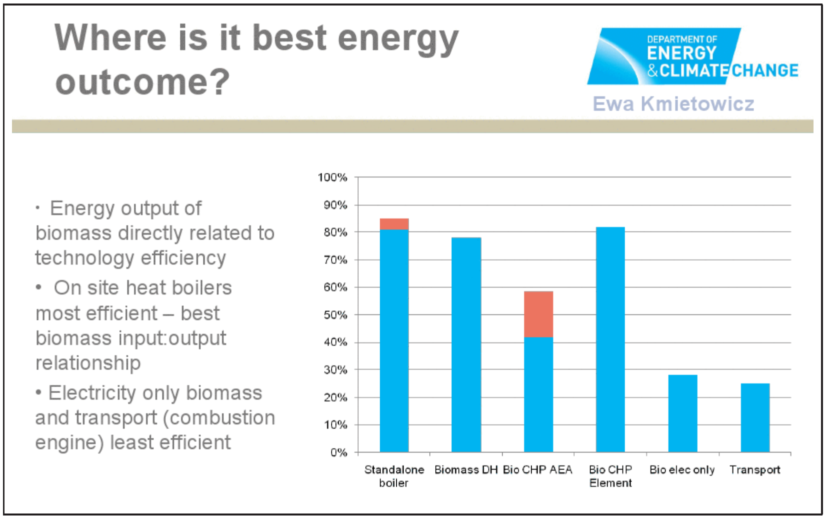


Figure 1.1. Illustrative page from a DECC document in which the metric of "efficiency" encourages the judgement that biomass-to-heat is better than biomass-to-electricity or biomass-to-work (transport). Courtesy of Ewa Kmietowicz.

> "It is more efficient to produce one unit of heat from biomass than one unit of electricity. The efficiency of a biomass boiler is around 80%; the efficiency of electricity generation is around 28% (Figure 1.1). So from a technical efficiency point of view it is better to put biomass into the heat sector."

I am not questioning these "efficiency" facts. Yes, you need around three times less biomass to produce one unit of heat than to produce one unit of electricity. But the **value judgement** that "it is **better** to put biomass into the heat sector" is founded on an implicit assumption – that "one unit of heat has the same value as one unit of electricity". *This is not correct.* If we were instead to judge one unit of electricity as having the same value as **four** units of heat, then, taking account of both efficiency and value, we would make the opposite recommendation:

> "Don't waste that tree! Electricity beats heat by 4 to 3!"

To make good policies, we need to unveil and revise these implicit assumptions. What *is* the value of heat?

When I use the word 'value', I should make clear straight away that I am not talking about the *financial* value of these alternative forms of energy today, which depends on people's desires and the technologies we have available. Rather, I am talking about the essential utility of energy, which scientists call by various names such as *exergy* or *available energy*.

Here is the fundamental idea: *you can easily turn one unit of electricity into one unit of heat, but you can't turn one unit of heat into one unit of electricity.* So one unit of heat has less value than one unit of electricity.

In *Sustainable Energy – without the hot air*, I tried to convey this idea by distinguishing between 'high grade' energy (electricity, kinetic energy, gravitational energy, chemical energy, mechanical work) and 'low grade'

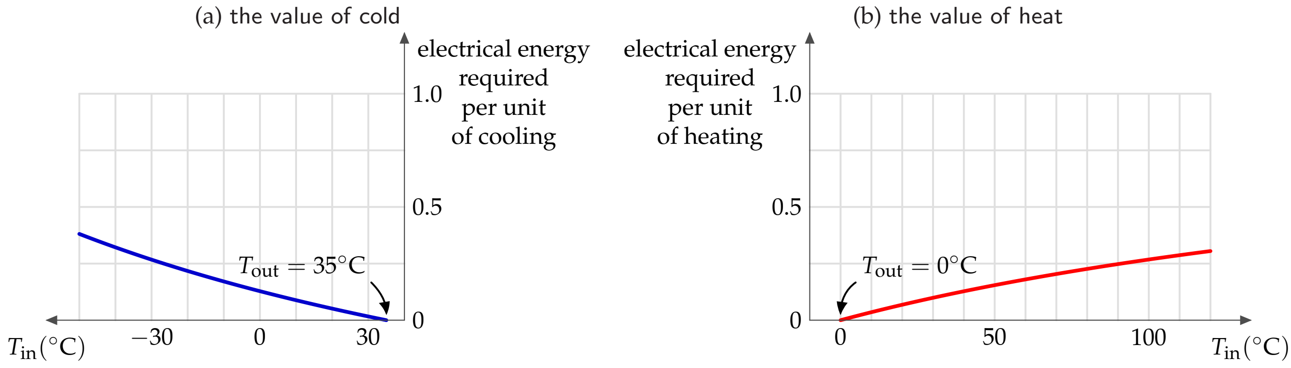


Figure 1.2. (a) the electrical energy required, according to the limits of thermodynamics, to pump heat *out* of a place at temperature T_{in} when the heat is being pumped to a place at temperature $T_{out} = 35\,°C$. The curve is

$$\frac{T_{out}}{T_{in}} - 1,$$

where both temperatures must be expressed on the kelvin scale ($0\,°C \simeq 273\,K$). The curve also shows the amount of electrical energy that could be generated from one unit of cold at temperature T_{in}. (b) the electrical energy required to pump heat *into* a place at temperature T_{in} when the heat is being pumped from a place at temperature $T_{out} = 0\,°C$. The curve is

$$1 - \frac{T_{out}}{T_{in}},$$

where both temperatures must be expressed on the kelvin scale.

energy (heat), but I didn't go into explicit detail. In this note I give more details – though I don't have space here to explain everything from first principles. One beauty of getting the details right is that we will end up with a single consistent way of valuing both heat and *cold*, which is a useful resource too.

The physicist defines the value of heat-energy in terms of how much high-grade energy it could be converted into, if we had ideal, perfect conversion machinery. All the high grade forms of energy are equivalent; idealised machines can be imagined that could convert, say, gravitational energy perfectly into electricity with a one-to-one exchange rate, or vice versa. Real-life hydroelectric facilities come quite close to this ideal exchange rate – one unit of electricity from each unit of potential energy. For heat, the exchange rate is not one-to-one. One unit of heat can be turned only into *less than* one unit of electricity. And, perhaps remarkably, the relationship is reversible, which means that one unit of electricity can deliver *more than* one unit of heat!

What precisely is this exchange rate? Well, the value of heat actually depends on the temperature T of the body containing the heat, and on the temperature of the surroundings, which I'll call "T_{out}". *The higher the temperature of the hot body, the greater the value of a unit of heat.*

Figure 1.2 shows part of figure E.13 from page 300 of the book, where I was discussing heat pumps. This figure shows how the value of one unit of heat depends on the temperature of the place to which it is delivered or from which it is taken. Figure 1.2b shows the value of heat. It shows the electrical energy required to deliver one unit of heat to a body that is at a temperature T_{in}, assuming the external temperature is $T_{out} = 0\,°C$. It also shows, conversely, the amount of electricity that could be generated *from* one unit of heat at temperature T_{in}, given ideal machinery. For example, the right-hand figure shows that a unit of heat from a body at $T_{in} = 90\,°C$ is worth 0.25 units of electricity. The curve is

$$1 - \frac{T_{out}}{T_{in}}, \tag{1.1}$$

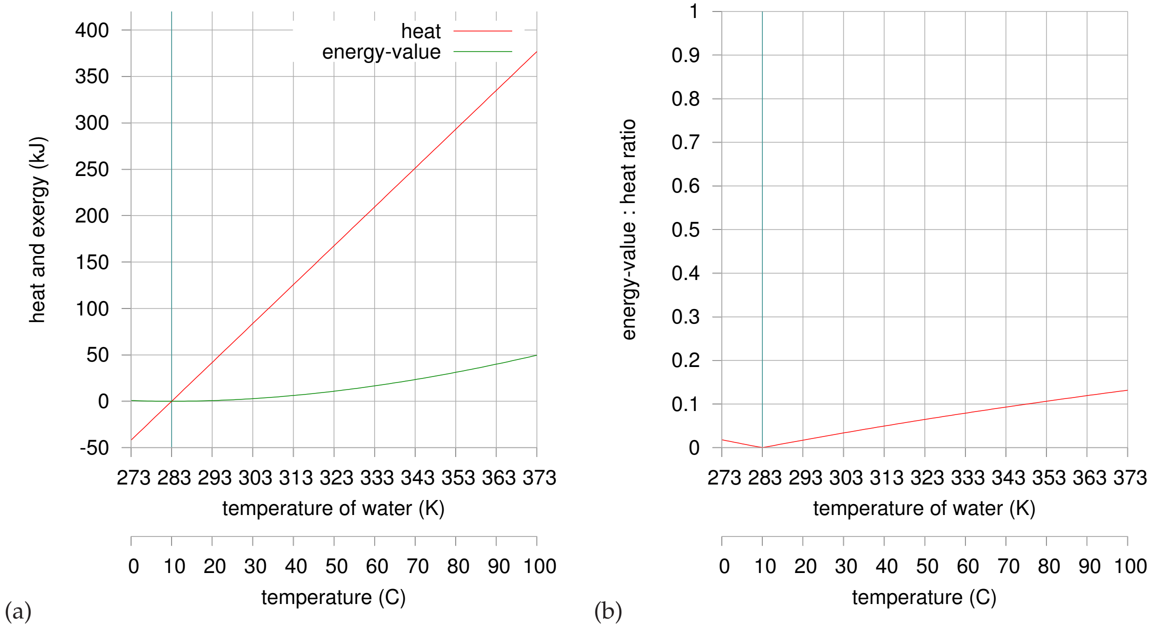


Figure 1.3. (a) The energy-value (or exergy) of warming up 1 kg of water, and the heat supplied, assuming the ambient temperature is $T_{\text{out}} = 10\,°\text{C}$, as a function of the temperature of the warmed water. (b) The ratio of the energy-value to the absolute value of the heat.

where both temperatures must be expressed on the kelvin scale ($0\,°\text{C} \simeq 273\,\text{K}$).

The value of warming something up or cooling it down

What we've discussed so far is the value of a little unit of heat arriving in a body – a room, say – that is already at a particular temperature T_{in}.

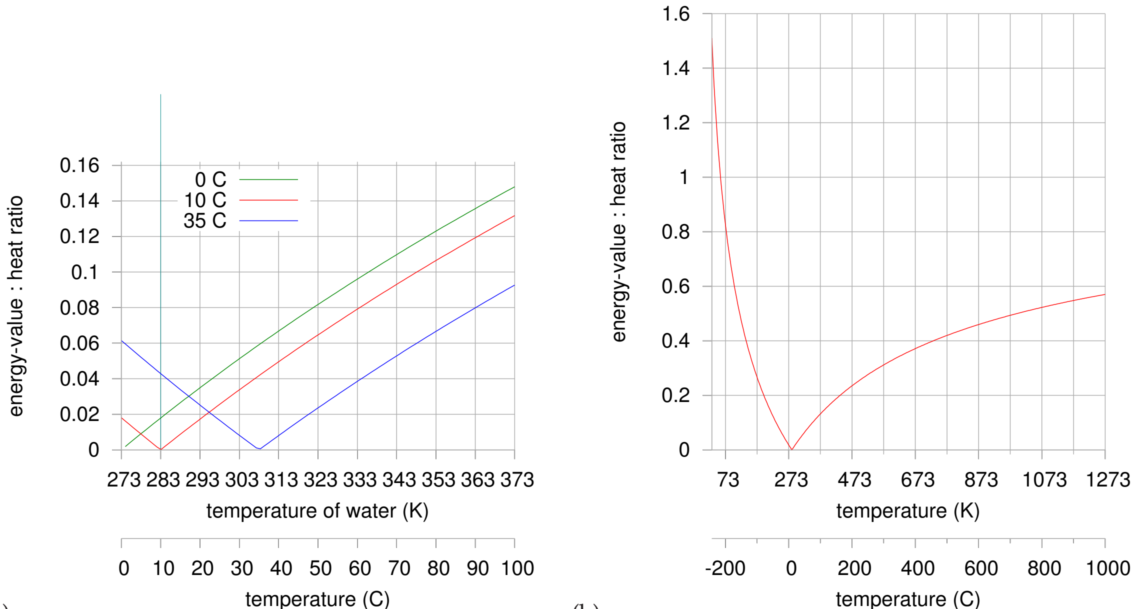
Now let's discuss the value of warming something up *from* the ambient temperature *to* a temperature T_{in}. We can imagine warming up some water from the mains temperature to $T_{\text{in}} \simeq 70\,°\text{C}$ in a sequence of little heat-delivery steps; at each step, the water is at some intermediate temperature T, and we give it a little heat, which, as described in the last section, has a value-per-unit-heat of

$$1 - \frac{T_{\text{out}}}{T}, \tag{1.2}$$

and the water's temperature T is bumped up a little. To work out the total value of a water-heating service, we simply add up all the values. (This sort of adding-up is called integration.) For the case of an object whose heat capacity is a constant, C, at all temperatures, the total energy-value comes out to:

$$C\left(T_{\text{in}} - T_{\text{out}}\right) - C T_{\text{out}} \ln\left(\frac{T_{\text{in}}}{T_{\text{out}}}\right). \tag{1.3}$$

This expression describes the amount of electrical energy that you could get from the heated object, given ideal machinery; and it describes the



(a) (b)

Figure 1.4. (a) The exergy–to–heat ratio for warming up water (or cooling it down) as a function of the final temperature, for three different ambient temperatures: 0 °C, 10 °C, and 35 °C. (b)

amount of electrical energy that would be required to warm up that heated object, given ideal machinery. If you don't like formulae, don't worry, just look at the pictures in figure 1.3. The first term $C\,(T_{\text{in}} - T_{\text{out}})$ can be recognized as the amount of heat delivered. The total energy-value in equation (1.3) is sometimes called the *exergy*.

Let's compare the energy-value of warming up the object with the heat supplied. Figure 1.3a compares these two quantities for the case of 1 kg of water, which has heat capacity $C = 4.2\,\text{kJ/K}$, assuming the ambient temperature is $T_{\text{out}} = 10\,°\text{C}$. Figure 1.3b shows the ratio of the energy-value to (the absolute value of) the heat. Figure 1.4a shows how this curve changes if the ambient temperature is turned down to 0 °C or up to 35 °C. Figure 1.4b shows the ratio (for $T_{\text{out}} = 10\,°\text{C}$ again) for a wider range of cold and hot temperatures. Notice that the most valuable heat of all is actually negative heat (i.e., cold), at very low temperatures!

1 kWh = 3600 kJ.

'Setting fire to fuel is a thermodynamic crime'

How does chemical energy fit into this discussion of heat and electricity? Well, as long as it's measured right, chemical energy has a 1:1 exchange rate with all the other forms of high-grade energy: electrical energy, kinetic energy, potential energy. Indeed this is almost a tautology because electrical energy is in fact a simple form of chemical energy – electrical current involves electrons changing their energy-level, just like many chemical reactions. Why did I say "chemical energy, *as long as it's measured right*"? Well, many properties of chemical reactions are tabulated, and only one

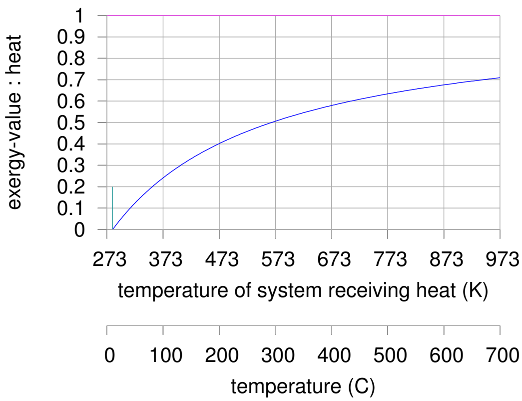


Figure 1.5. Exergy-efficiency of setting fire to fuel. This is the same graph as in figure 1.2b, for a wider range of temperatures. I emphasize that the *conventional* efficiency (the ratio of heat out to chemical energy in) is 1. This graph shows the "true" efficiency in terms of the ratio of value-out to value-in.

of these properties is the correct energy-value for our purposes – the *exergy*. Usually the *Gibbs free energy* of a reaction tells you the exergy that we want. But often people focus instead a measure of the heat emitted by a reaction, such as the *enthalpy*, the *high heat value*, or the *low heat value*. For many reactions involving combustion of fuel, the Gibbs free energy is quite well approximated by the *enthalpy*, which is the same as the *high heat value*. I therefore recommend using the *high heat value* as the measure of energy-value of a fuel, if the Gibbs free energy is not to hand.

Now, after we set fire to a fuel, what we obtain is the heat of combustion, delivered at the temperature of the boiler, call it T_{in}. So if we started with one unit of high-grade energy, the value that we have now is:

$$1 - \frac{T_{\text{out}}}{T_{\text{in}}}. \quad (1.4)$$

This is the "energy-value efficiency" of setting fire to fuel, when we measure value using exergy. (As before, all temperatures in this formula are on the kelvin scale.)

Figure 1.5 shows this true efficiency or exergy-efficiency for a range of boiler temperatures from 0 °C to 700 °C, assuming an ambient temperature of $T_{\text{out}} = 10\,°\text{C}$.

A key thing to notice is that setting fire to fuel substantially reduces the value of its energy, unless the fire is extremely hot. (That's why power station designers are always pushing their boilers and turbines to work at higher temperatures.) If the temperature of the boiler is less than 250 °C, then setting fire to the fuel instantly loses more than half of the value. If the temperature is less than 100 °C, more than 70% of the value is lost. This is what I mean when I say, jokingly, that setting fire to fuel to deliver low-temperature heat is a thermodynamic crime.

Recommendations

We use several criteria to evaluate energy options – price, and environmental impact, for example. I recommend that "exergy efficiency" (exergy-out divided by exergy-in) should take the place of the conventional "efficiency" (energy-out divided by energy-in) on this list of criteria.

Further applications

This recommendation is particularly relevant to heat pumps and to combined heat and power. On pages 148–150 of *Sustainable Energy – without the hot air*, I compared combined heat and power solutions in terms of "electrical efficiency" and "heat efficiency"; what I really would have liked to display on the vertical axis was not "heat efficiency" but exergy-efficiency. I hope to elaborate on this in the future.